

DIFFUSIVITY AND DYNAMICS OF LINEAR INTERNAL WAVES FROM A LAGRANGIAN PERSPECTIVE

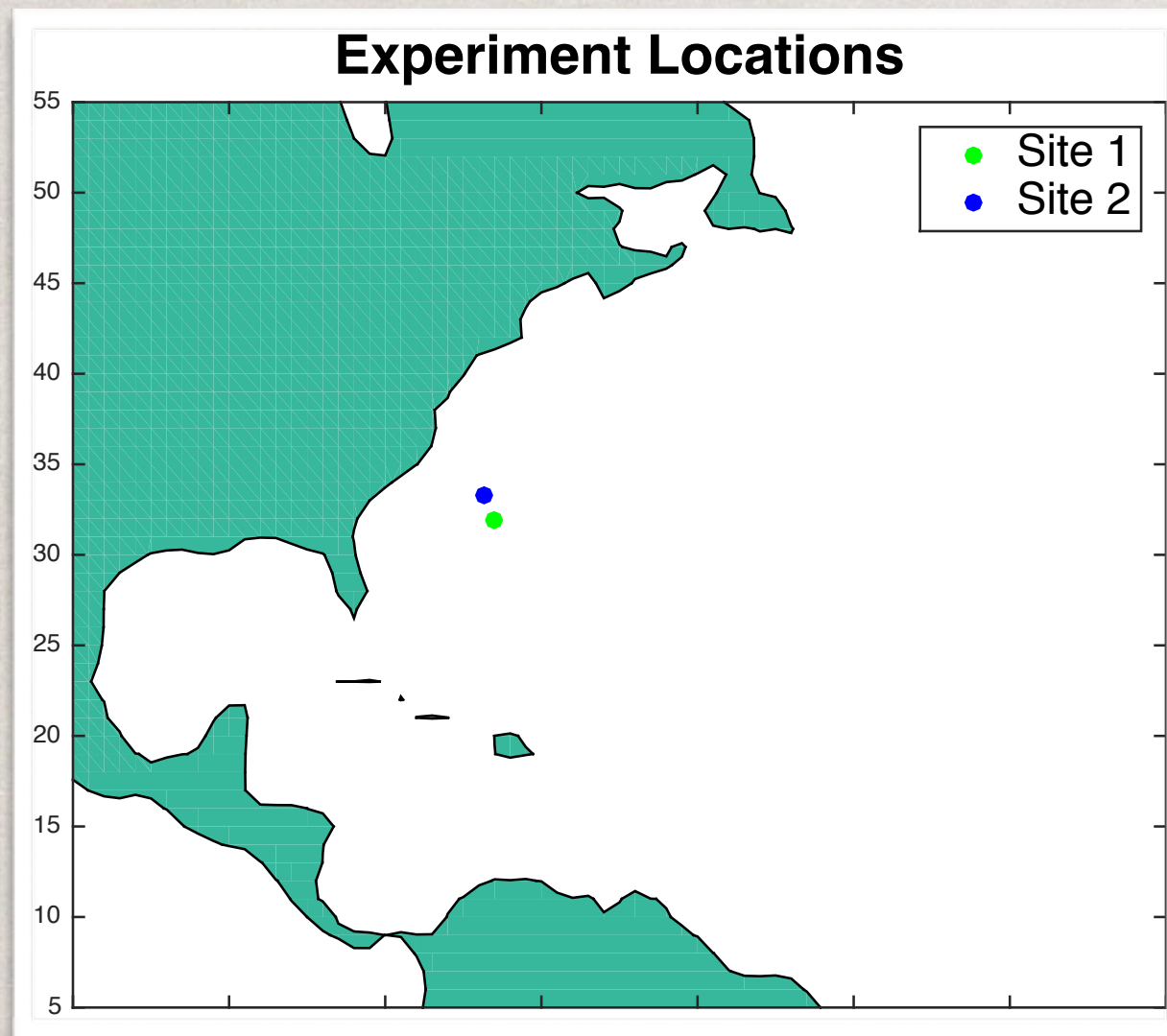
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LATMIX GROUP

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☼ What causes the $O(1)$ m²/s lateral diffusivity at scales 0.1 to 10 km?



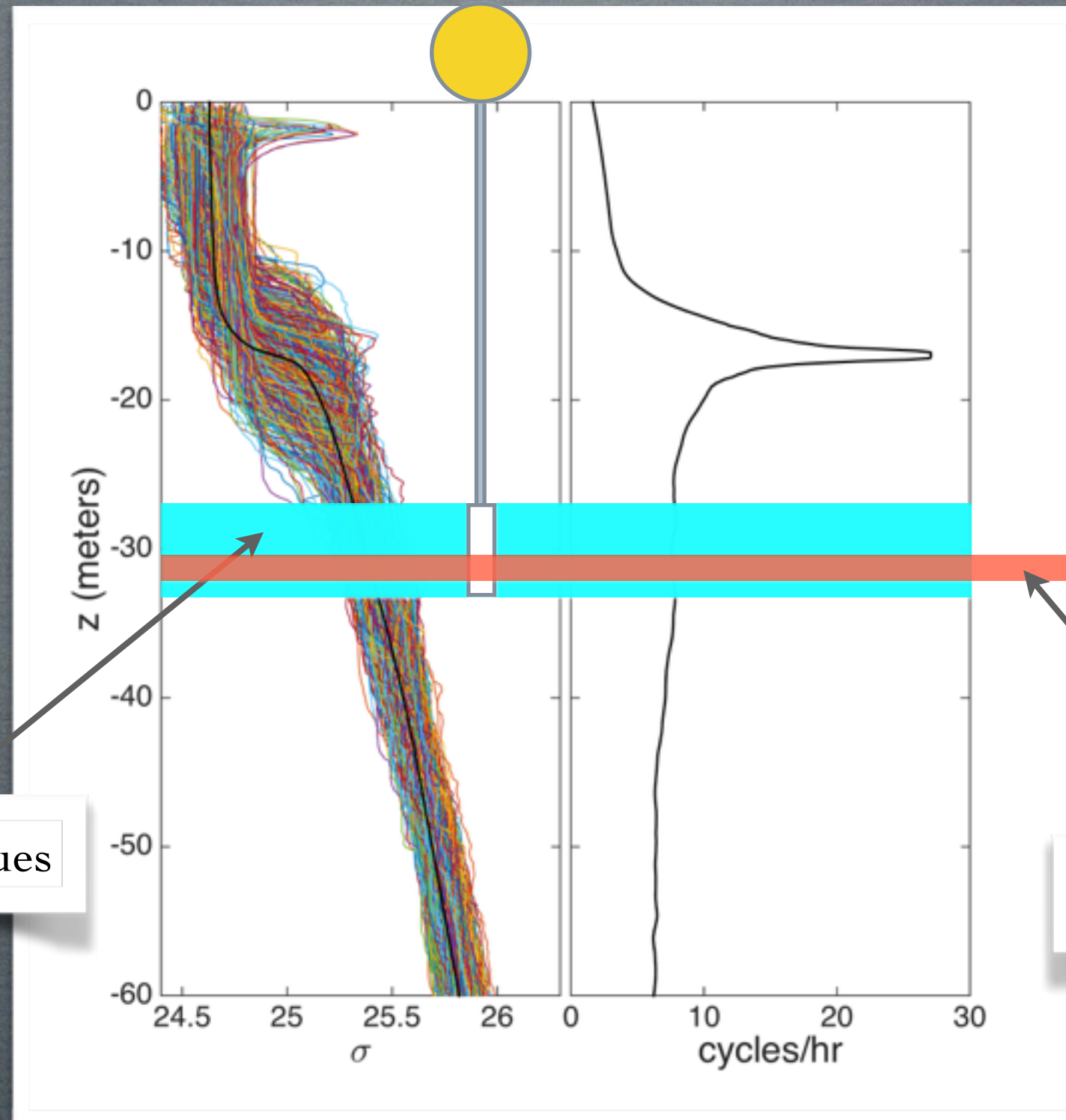
Measurements from

- dye release
- gliders
- drifters
- many others...

Experiment over 6 days

DENSITY FROM GLIDER PROFILES

KIPP SHEARMAN



Drifter drogues

Dye injection

PRIMARY RESULTS

(AFTER REMOVING STRAIN)

Site 1 (weak strain):

✱ Dye diffusivity: $O(1) \text{ m}^2/\text{s}$

✱ Drifters diffusivity: $O(.1) \text{ m}^2/\text{s}$

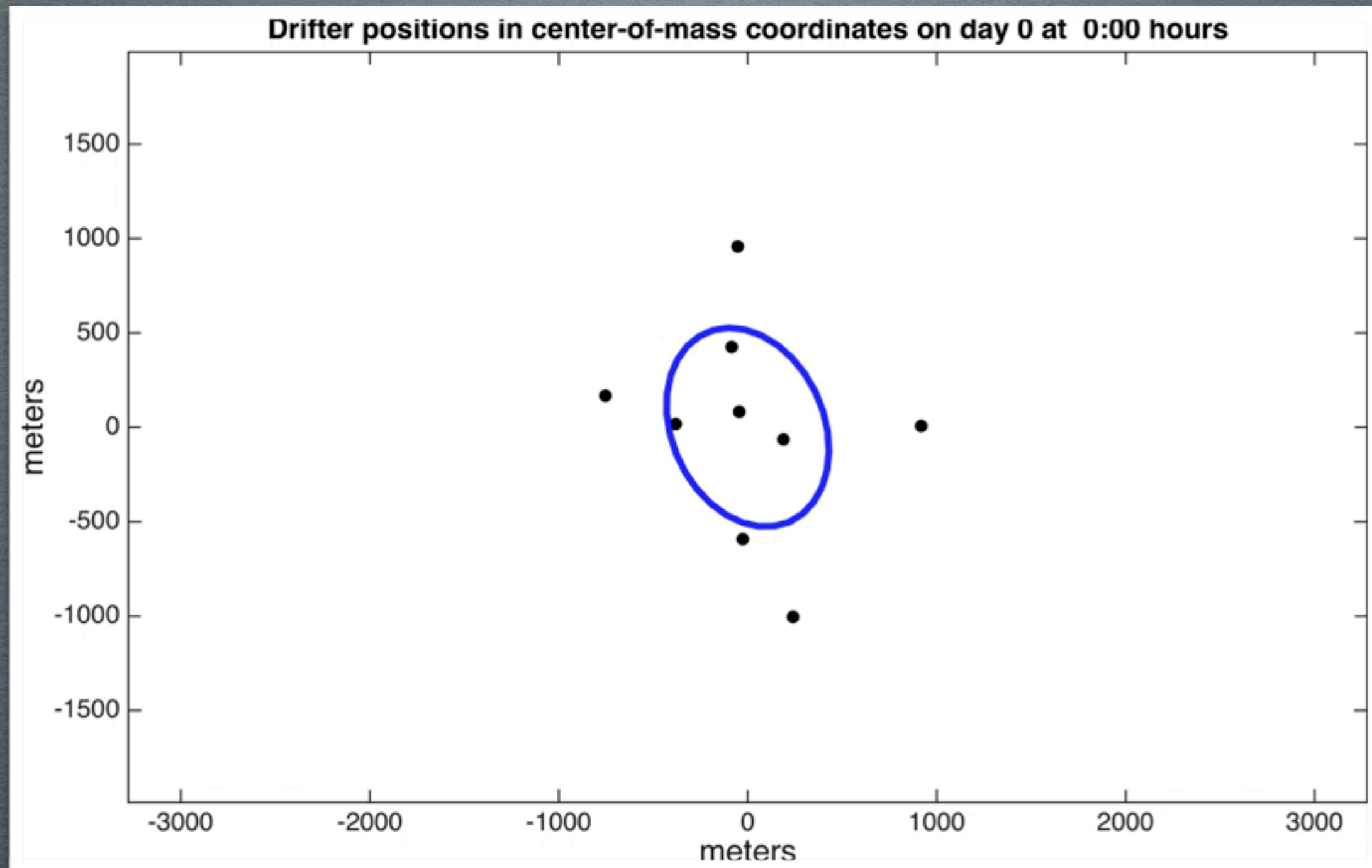
Site 2 (moderate strain):

✱ Dye diffusivity: $O(1) \text{ m}^2/\text{s}$

✱ Drifters diffusivity: $O(1) \text{ m}^2/\text{s}$

Drifters from both sites show a Garrett-Munk spectrum

SITE 1 DRIFTER TRAJECTORIES IN CENTER-OF-MASS COORDINATES.



 Observed 2nd moment

 Isotropic model

STRAIN-DIFFUSIVITY MODEL

- ✱ Isotropic diffusivity is *not* a good model.
- ✱ The drifters are clearly being stretched by a strain field.
- ✱ Need a model to account for strain and diffusivity.

$$u_B(t) = u_0(t) + u_x x + u_y y$$

$$v_B(t) = v_0(t) + v_x x + v_y y$$

VORTICITY-STRAIN-DIFFUSIVITY MODEL

✻ Model for tracer $\phi(x, y, t)$,

$$\begin{aligned} \phi_t + \left(u_0(t) + \frac{\sigma_n}{2}x + \frac{1}{2}(\sigma_s - \zeta)y \right) \phi_x \\ + \left(v_0(t) + \frac{1}{2}(\sigma_s + \zeta)x - \frac{\sigma_n}{2}y \right) \phi_y = \kappa \nabla^2 \phi \end{aligned}$$

✻ Model for particle $(x_i(t), y_i(t))$,

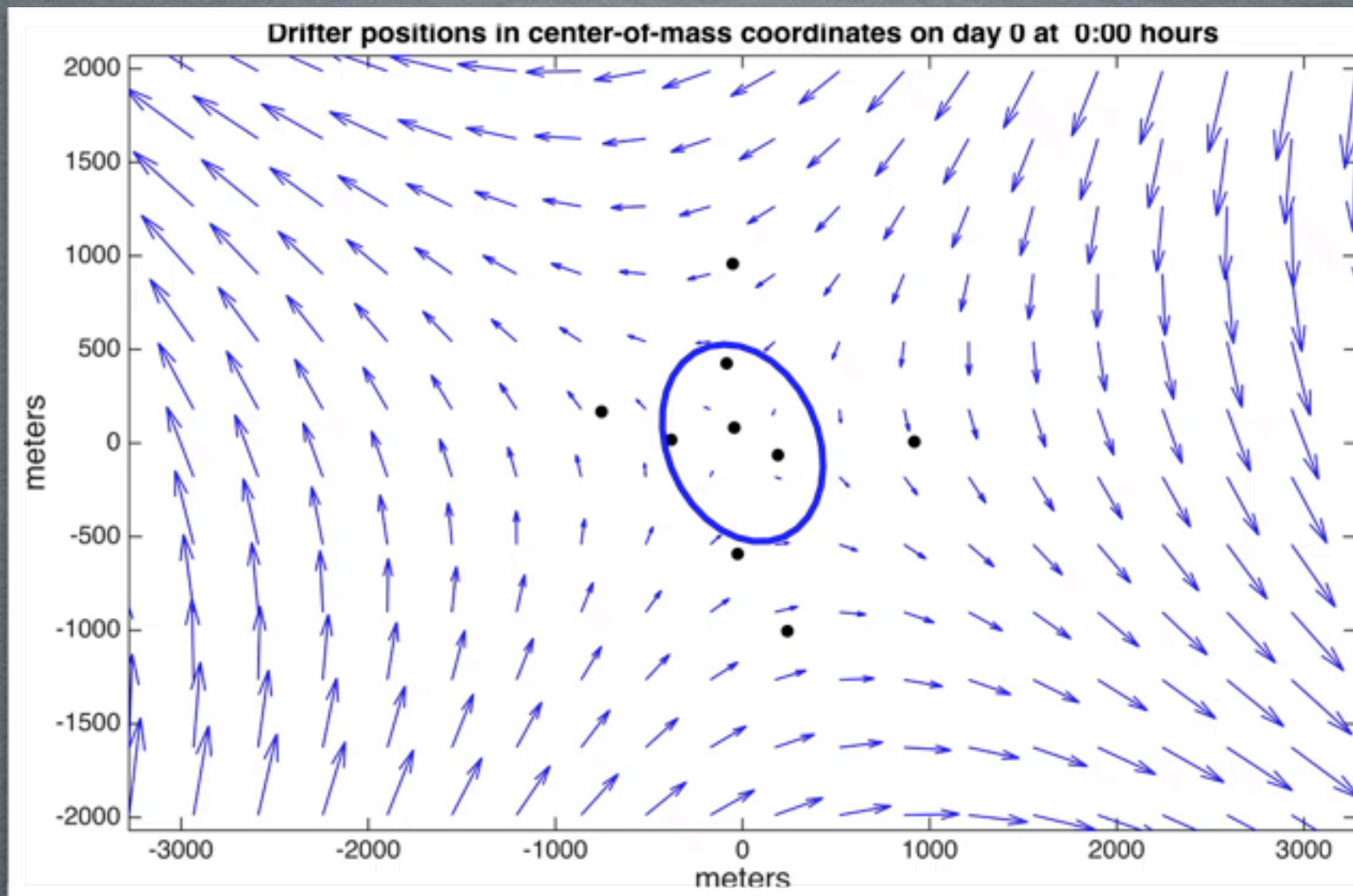
$$\begin{aligned} \partial_t \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \sigma_n & \sigma_s - \zeta \\ \sigma_s + \zeta & -\sigma_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \\ + \begin{bmatrix} u_0(t) \\ v_0(t) \end{bmatrix} + \sqrt{2\kappa} \mathbf{dW}_t \end{aligned}$$

✻ Vorticity ζ , strain σ , angle θ , diffusivity κ , are parameters to find by minimizing an error function.

A HIERARCHY OF MODELS

1. Fit drifters to three models,
 - I. diffusive (κ)
 - II. strain-diffusive (σ, θ, κ)
 - III. vorticity-strain-diffusive ($\zeta, \sigma, \theta, \kappa$)
2. Generate synthetic data to test for statistical significance of the fits.

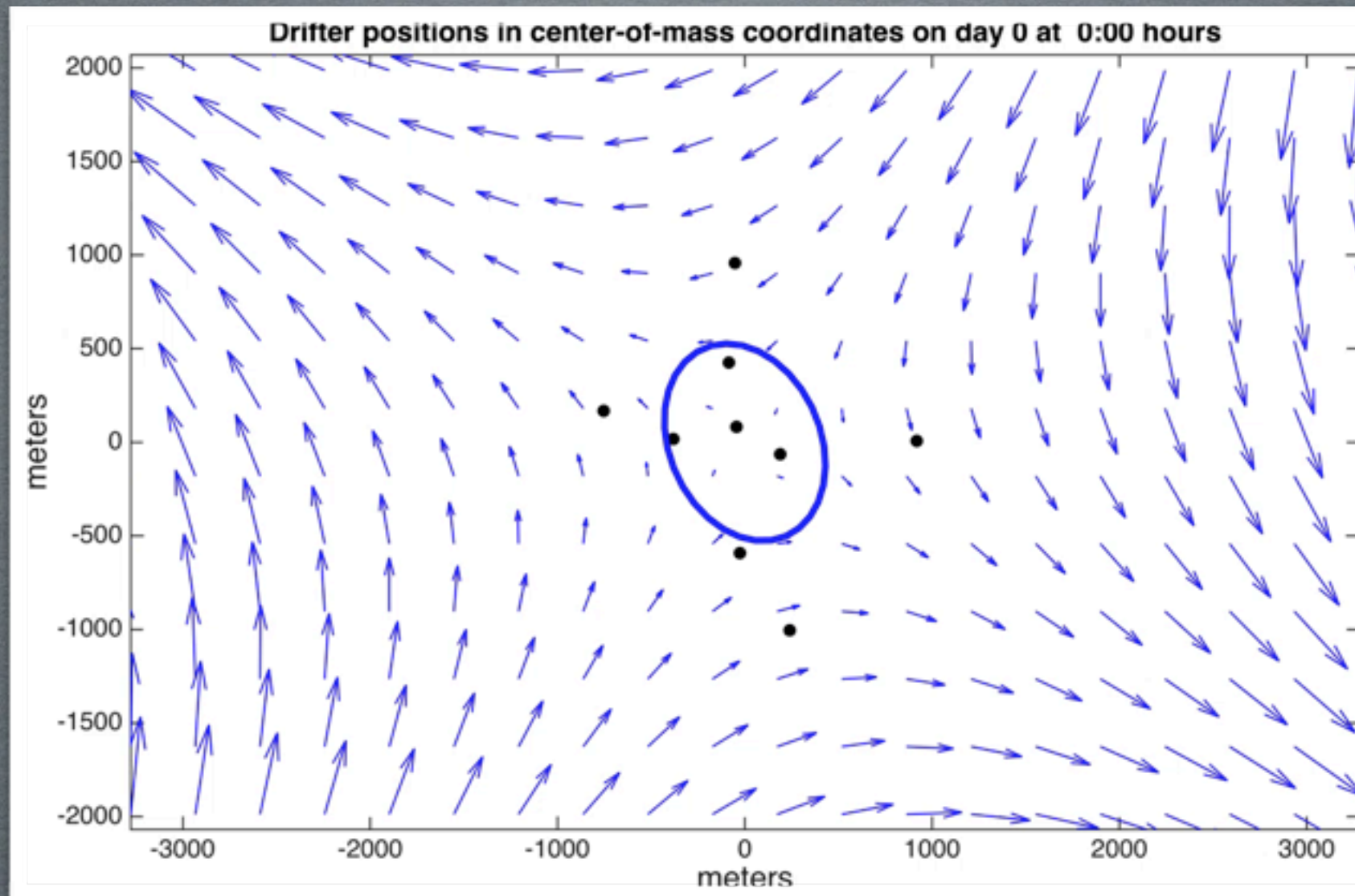
DRIFTER TRAJECTORIES IN CENTER-OF-MASS COORDINATES.



 Observed 2nd moment

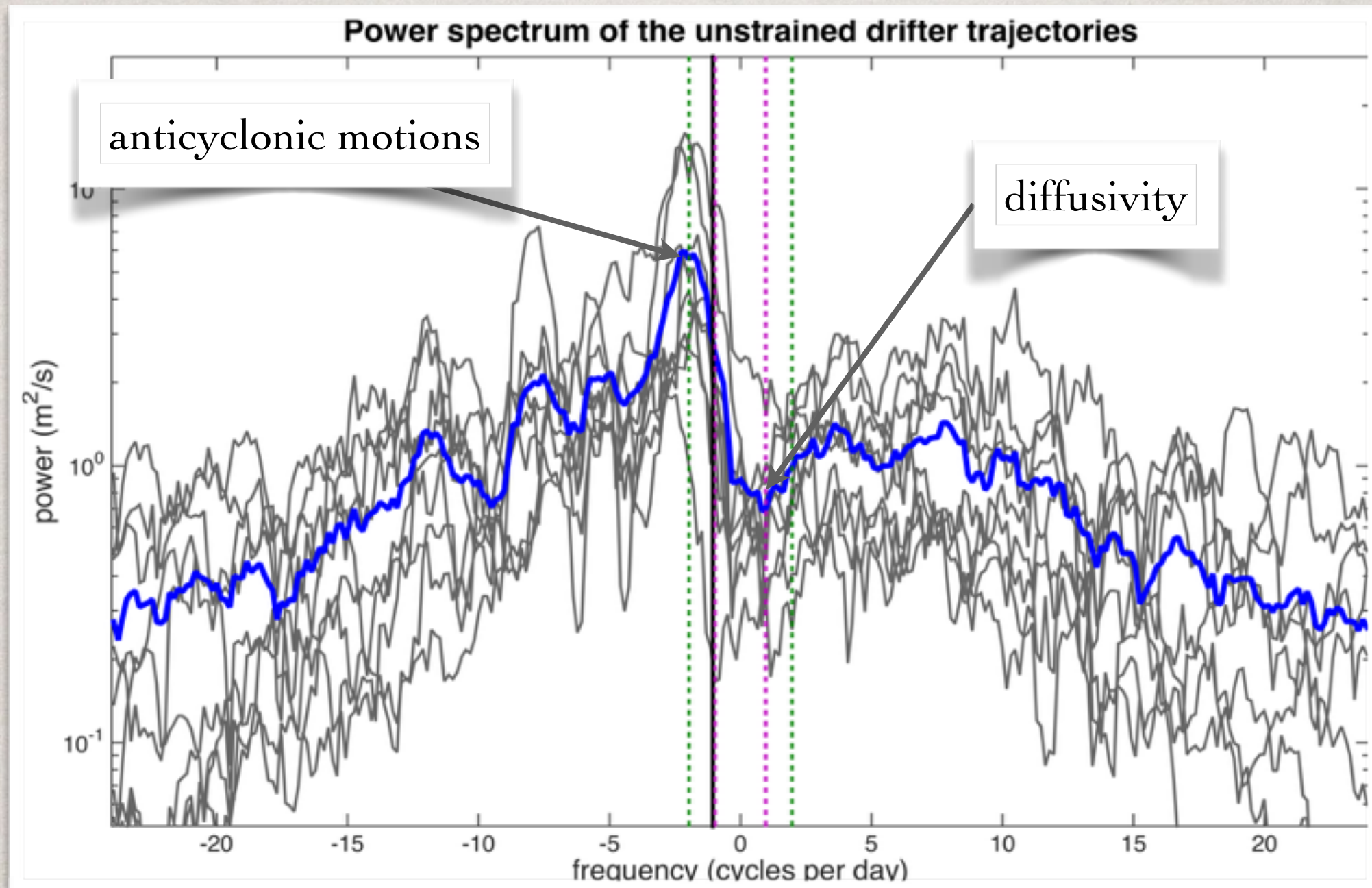
 Strain-diffusivity model,
 $\kappa = 0.18 \text{ m}^2/\text{s}, \sigma = 3.52 \cdot 10^{-6} \text{ s}^{-1}, \theta = -33^\circ$

SYNTHETIC TRAJECTORIES IN CENTER-OF-MASS COORDINATES.



Stochastic trajectories **look** different

VELOCITY SPECTRA

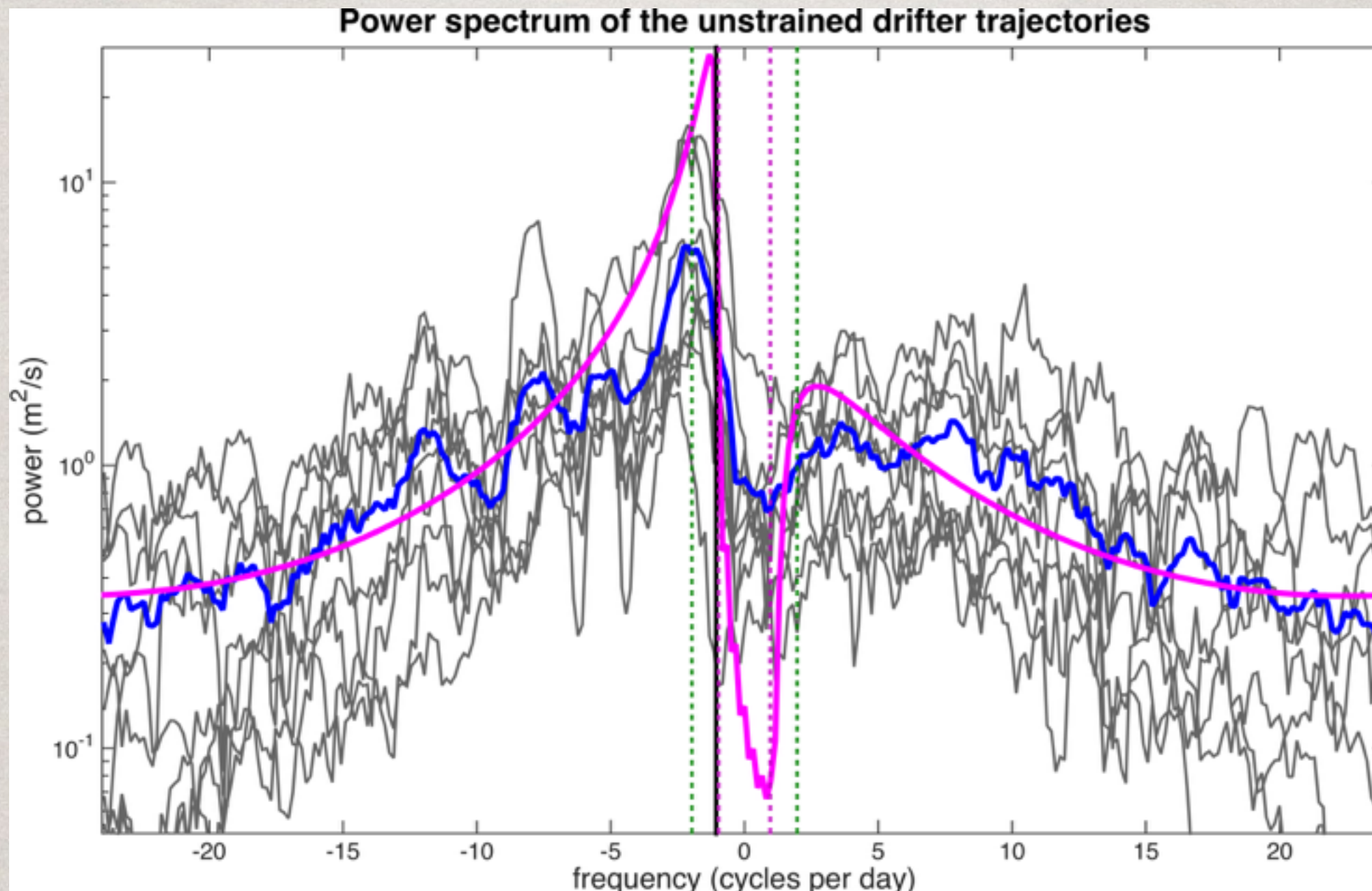


- Velocity spectrum looks like internal waves.
- Single particle diffusivity estimates also give $\kappa = 0.2 \text{ m}^2/\text{s}$

INTERNAL WAVE VELOCITY SPECTRUM

- ✱ Garrett-Munk spectrum is a model for the internal wave spectrum.
- ✱ Need a 'Lagrangian' version — same as Eulerian version to lowest order.
- ✱ Need to center-of-mass frame attenuation filter.

SITE 1 SPECTRA



- ✿ Theoretical GM spectrum for center-of-mass internal waves at 0.5 GM reference level (magenta).

LINEAR INTERNAL WAVES

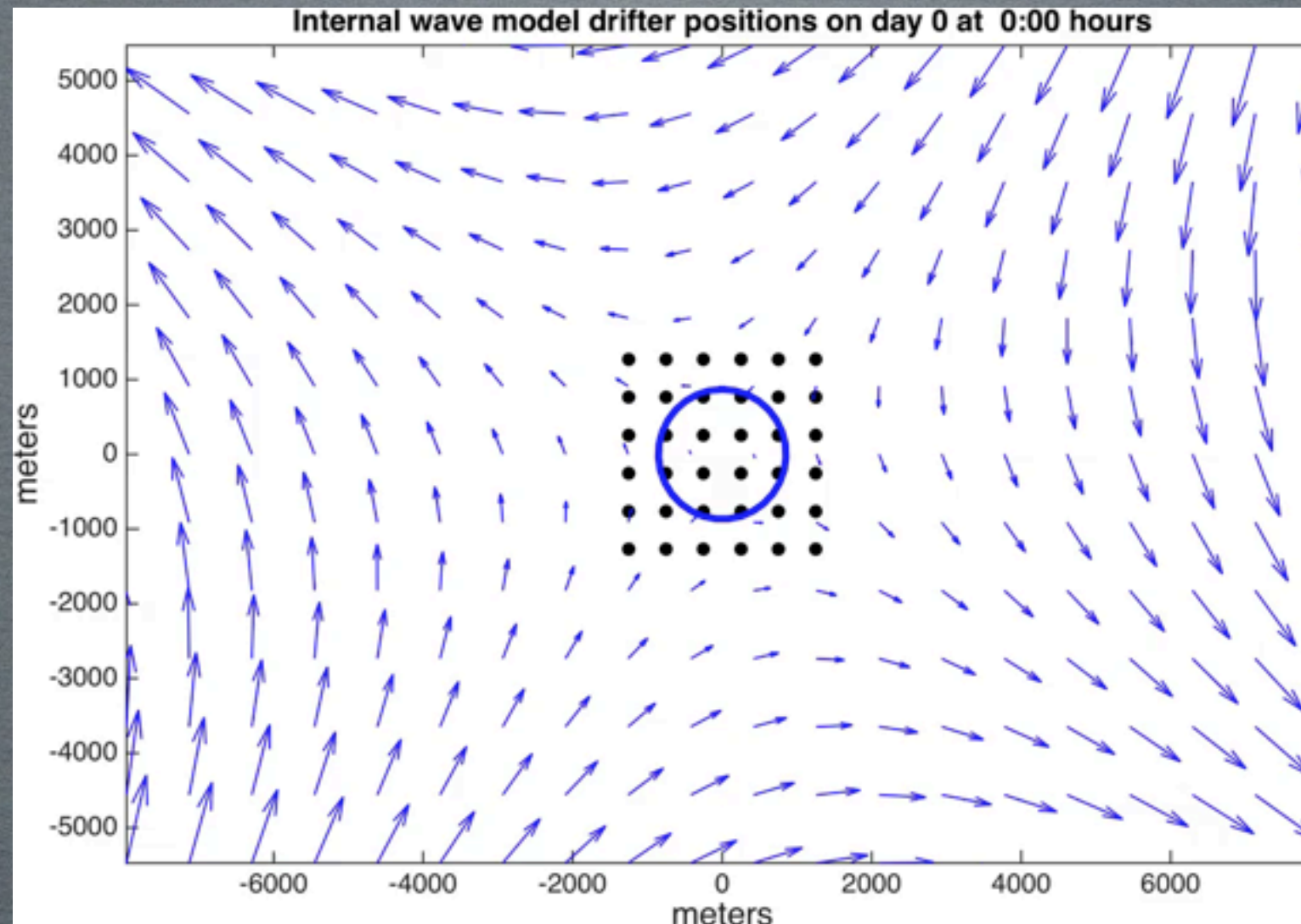
Can the observed spectra and diffusivity be explained by linear internal waves?

INTERNAL WAVE MODEL

- ✱ Uses realistic stratification.
- ✱ Matches energy level at 50 meters with observations.
- ✱ Model uses *analytical* solution to advect particles.
- ✱ Vertical resolution of 1 meter, horizontal resolution of 60 meters.

DRIFTER-LIKE PARTICLES

FROM AN INTERNAL WAVE MODEL WITH STRAIN

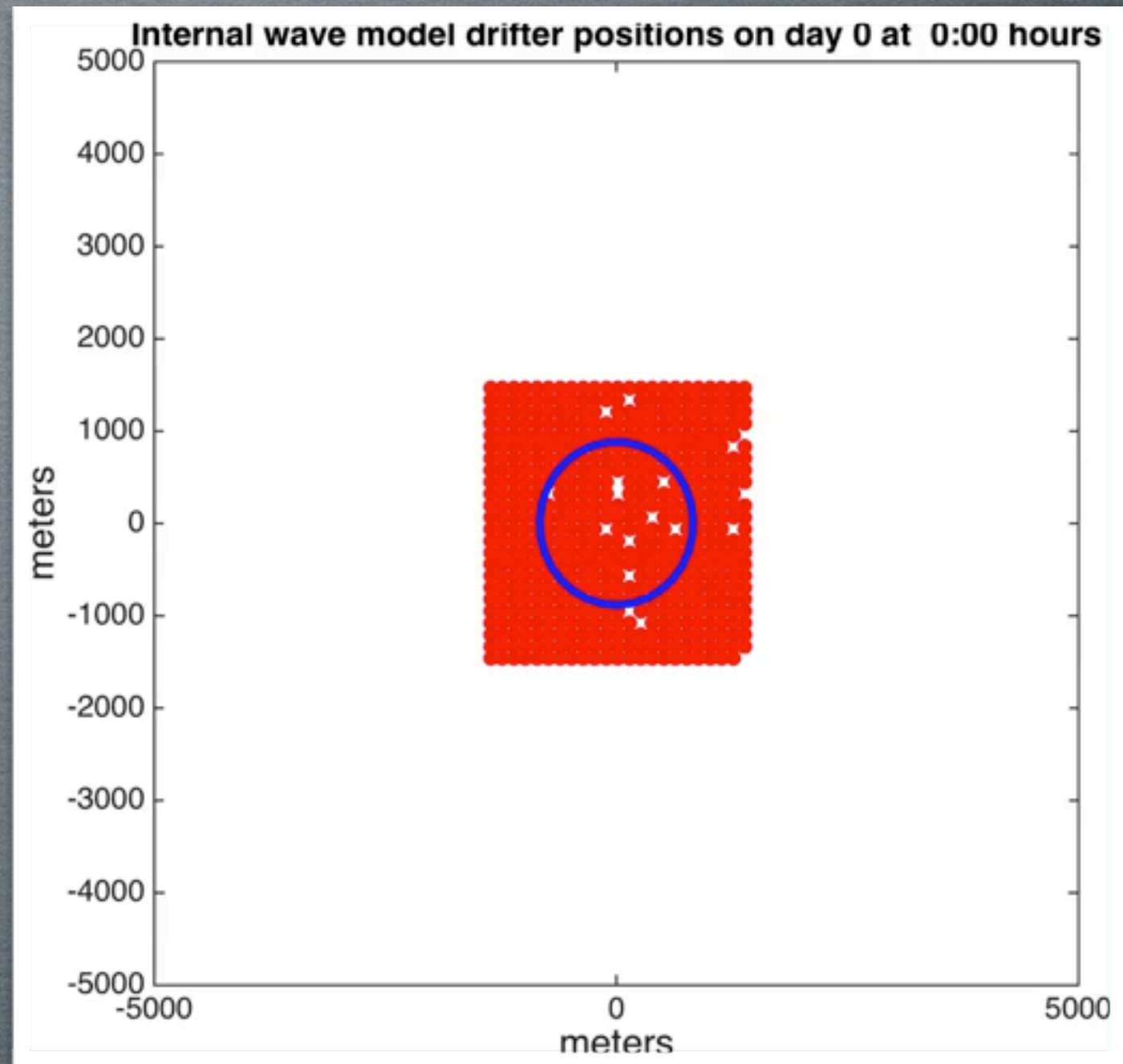


○ Observed 2nd moment

○ Strain-diffusivity model,
 $\kappa = 0.18 \text{ m}^2/\text{s}, \sigma = 3.52 \cdot 10^{-6} \text{ s}^{-1}, \theta = -33^\circ$

DYE-LIKE PARTICLES

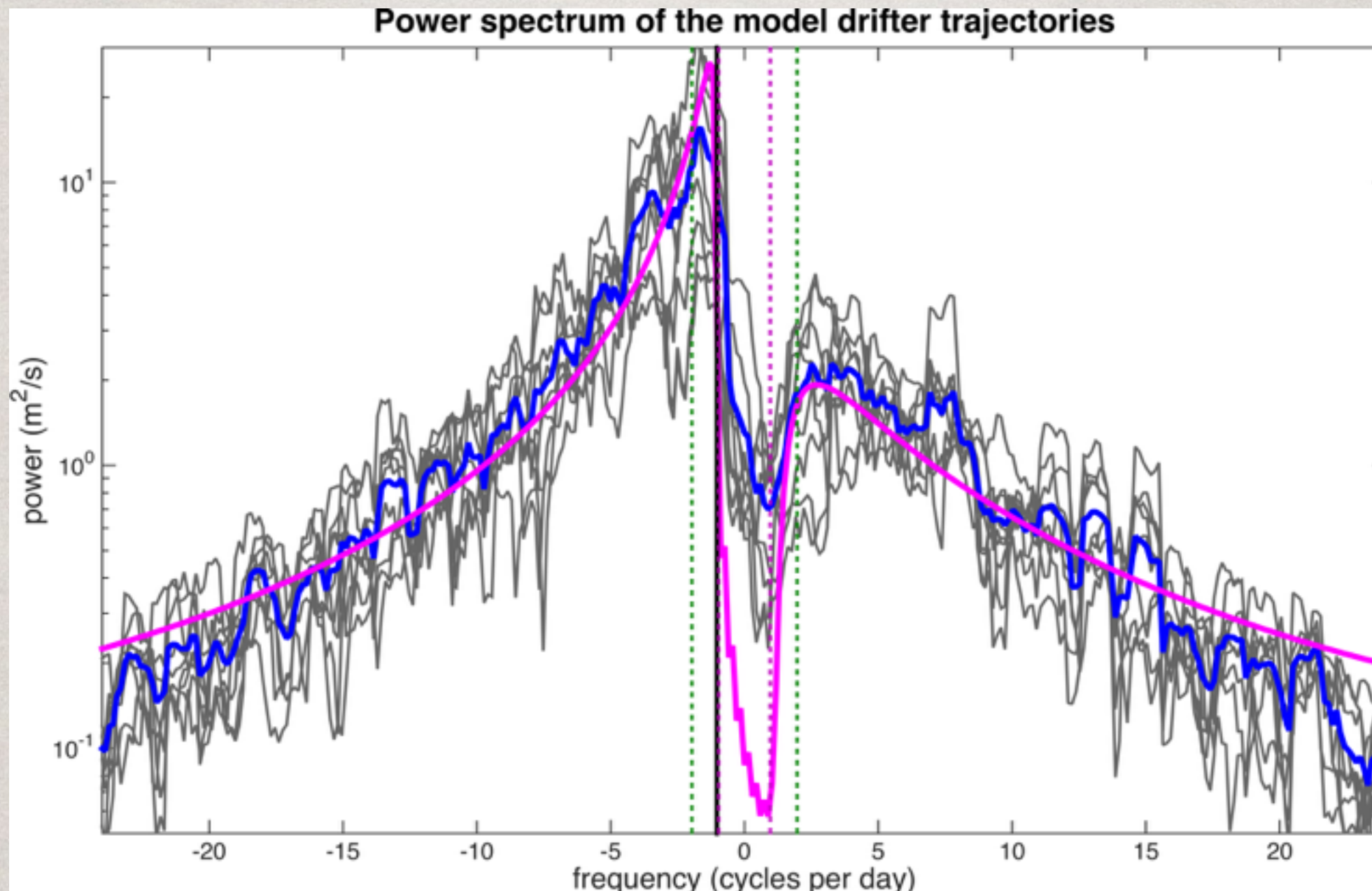
- Much higher diffusivity than the drifter-like particles
- Diffusive and non-diffusive particles are nearly indistinguishable which means shear-dispersion is a negligible effect.



○ Observed 2nd moment

○ Isotropic model, $\kappa = 1.0 \text{ m}^2/\text{s}$

VELOCITY SPECTRA



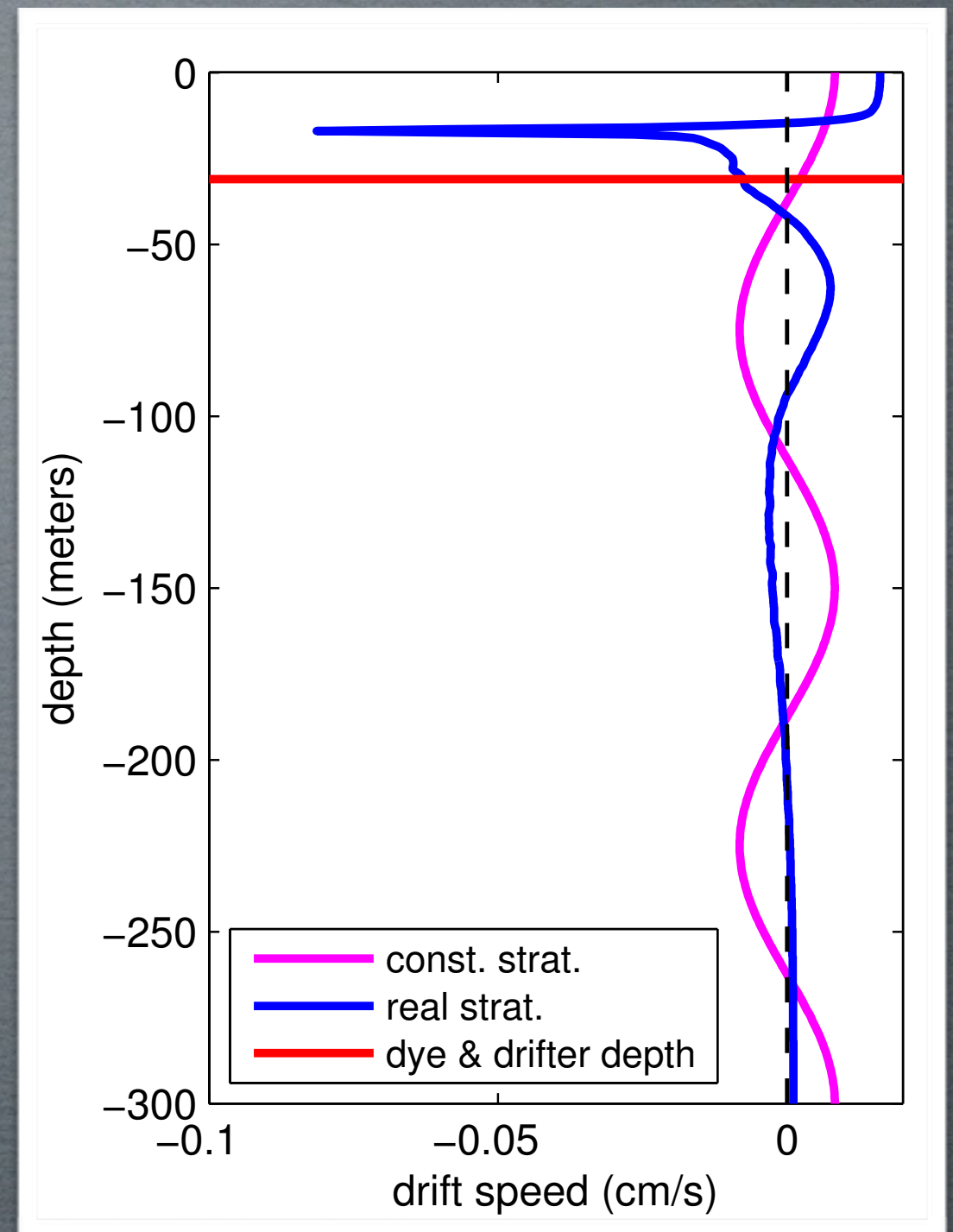
- ✿ Theoretical GM spectrum for center-of-mass internal waves at 0.5 GM reference level (magenta).

STOKES DRIFT

- ✱ Linear internal wave model leaves exactly one mechanism for dispersion: **Stokes drift**.
- ✱ Previous estimates of Stokes drift by internal waves suggest a diffusivity much too small.
- ✱ So what's different?

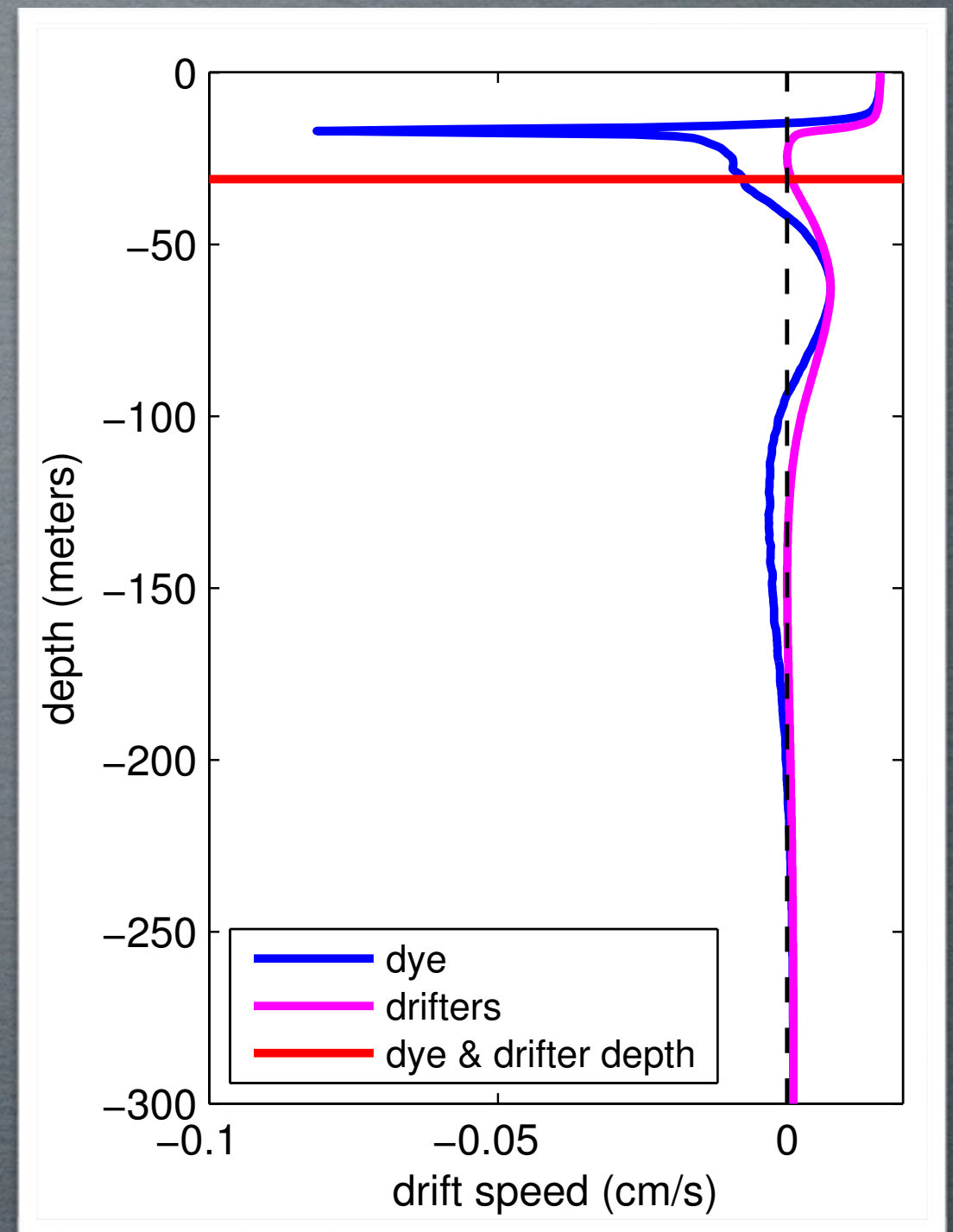
STOKES DRIFT AS A FUNCTION OF DEPTH

- Vertical mode-2, 1 cm/s, 1.5 km wavelength wave.
- Pycnocline enhances Stokes drift, just like the ocean surface.



STOKES DRIFT AS A FUNCTION OF DEPTH

- Comparison of drifter versus dye drift.
- Dye dominant, depending on depth.



CONCLUSIONS

- ✱ Drifters consistently show a much lower diffusivity than the dye release.
- ✱ The difference may be explained by their different transport mechanisms in realistic stratification.
- ✱ Shear dispersion does not appear to be significant.
- ✱ Observed diffusivity is consistent with the hypothesis that the dynamics are controlled by linear internal waves.
- ✱ Stokes drift may account for the observed diffusivity.

LINEAR INTERNAL WAVES

✻ Single internal wave,

$$u(x, y, z, t) = U \cos(kx + \omega t) F_j(z)$$

$$v(x, y, z, t) = -U \frac{f}{\omega} \sin(kx + \omega t) F_j(z)$$

$$w(x, y, z, t) = U h_j k \sin(kx + \omega t) G_j(z)$$

$$\eta(x, y, z, t) = -U h_j \frac{k}{\omega} \cos(kx + \omega t) G_j(z)$$

✻ Vertical structure determined from eigenvalue problem,

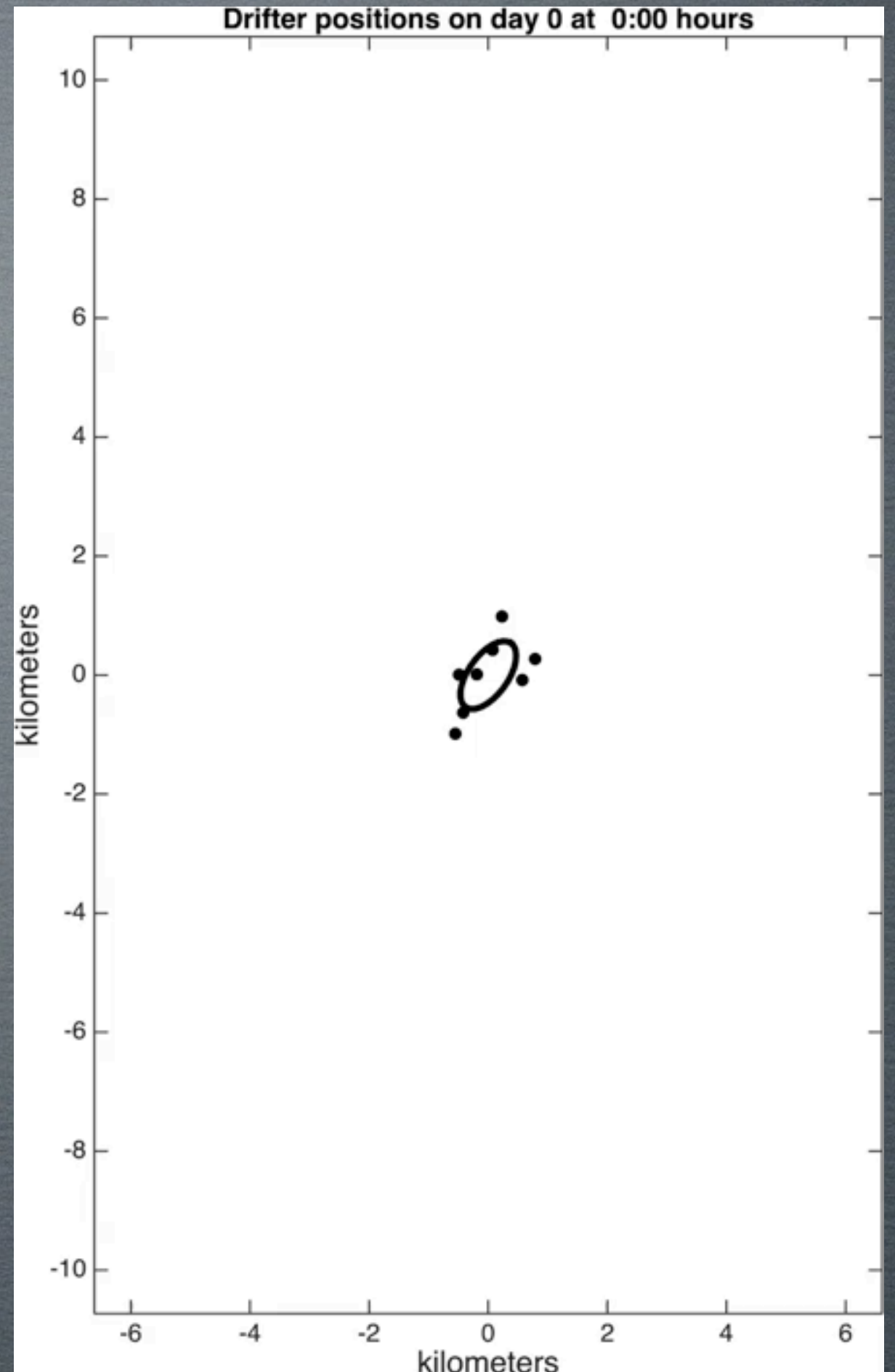
$$G_{zz}^{(j)}(z) - k^2 G^{(j)}(z) = \frac{f_0^2 - N^2(z)}{g h_j} G^{(j)}(z)$$

$$F(z) = h G_z(z)$$

✻ Now sum up a full Garrett-Munk like spectrum

SITE 2 DRIFTER TRAJECTORIES

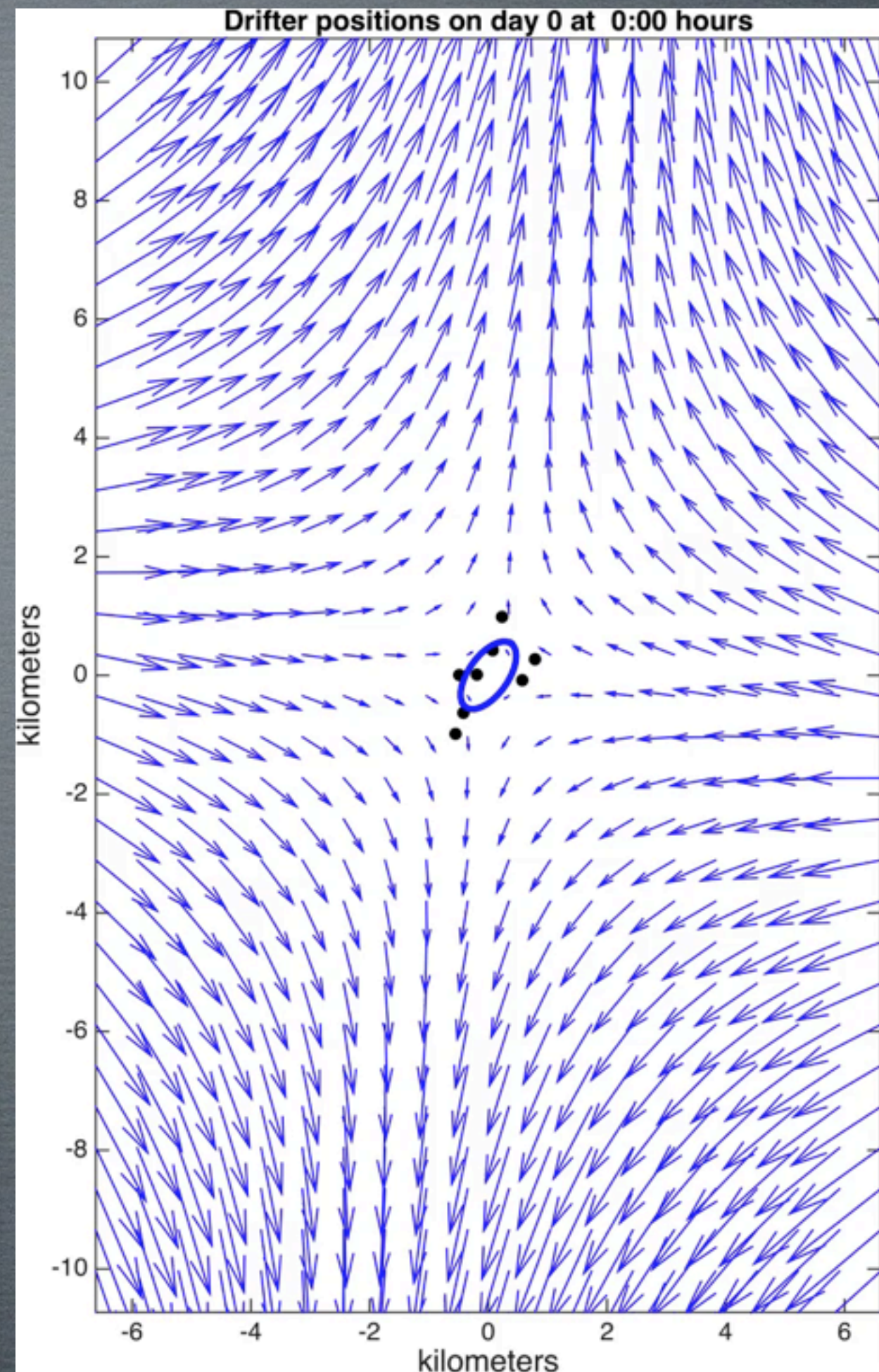
Moderate time varying
strain field.



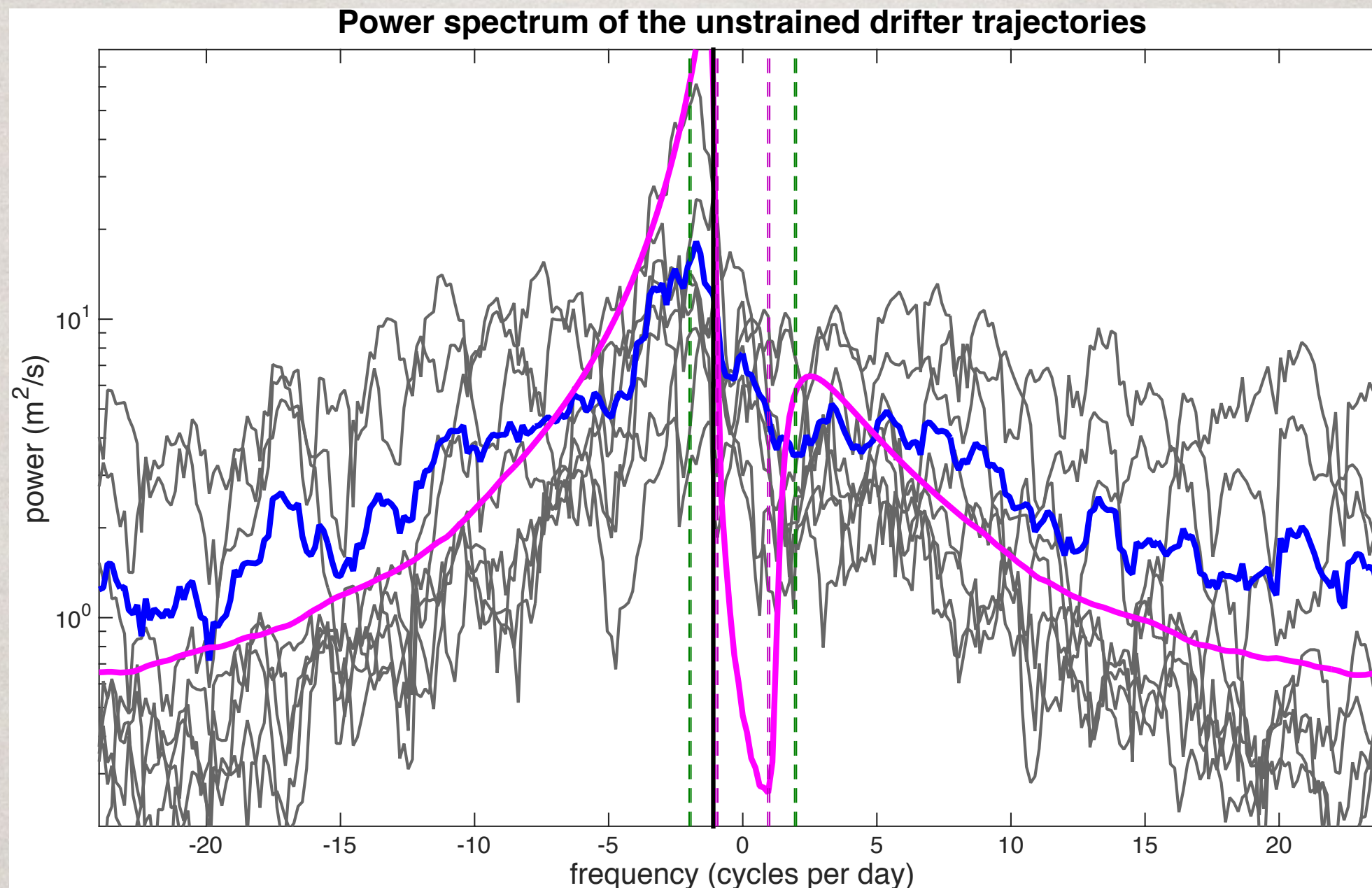
SITE 2 TRAJECTORIES WITH SOLUTION

 Observed 2nd moment

 Strain-diffusivity model
 $\bar{\kappa} = 1.1 \text{ m}^2/\text{s}$



SITE 2 SPECTRA



- ☼ Theoretical GM spectrum for center-of-mass internal waves at 1.0 GM reference level (magenta).

DRIFTER-LIKE PARTICLES

FROM A LINEAR INTERNAL WAVE MODEL

- More particles than the real experiment, but similar scales.

 Observed 2nd moment

 Isotropic model, $\kappa = 0.05 \text{ m}^2/\text{s}$

