

Inertial Oscillations & Eddies

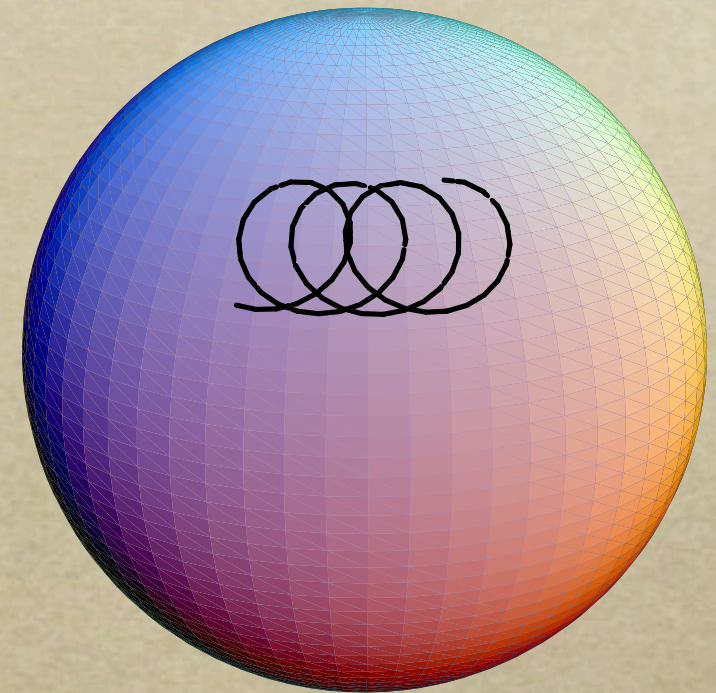
Jeffrey J. Early

Outline

- *Point mass on the Earth*
- *Disk on the Earth*
- *Quasigeostrophic Eddies*

Inertial Oscillations

What is the motion of an unforced particle on the surface of the earth?



The Experiment

*Roll a ball on the ground – uninterrupted by other forces
– what path does it take?*

Inertial Oscillations

Objective

What would an accelerometer trapped in an inertial oscillation measure?

The Experiment

- *Build a float with an accelerometer and a way to record the data.*
- *Drop the float in the ocean.*
- *If the float were to get trapped in an a 'pure' inertial oscillation, what would the accelerometer read?*

Inertial Oscillations

Particle at rest

$$\mathbf{a}_f = 0$$

Particle at rest, rotating frame

$$\mathbf{a}_r + 2\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) = 0$$

Inertial Oscillations

Equations of motion in rotating coordinates

$$\mathbf{a}_r + 2\boldsymbol{\omega} \times \mathbf{v}_r = 0$$

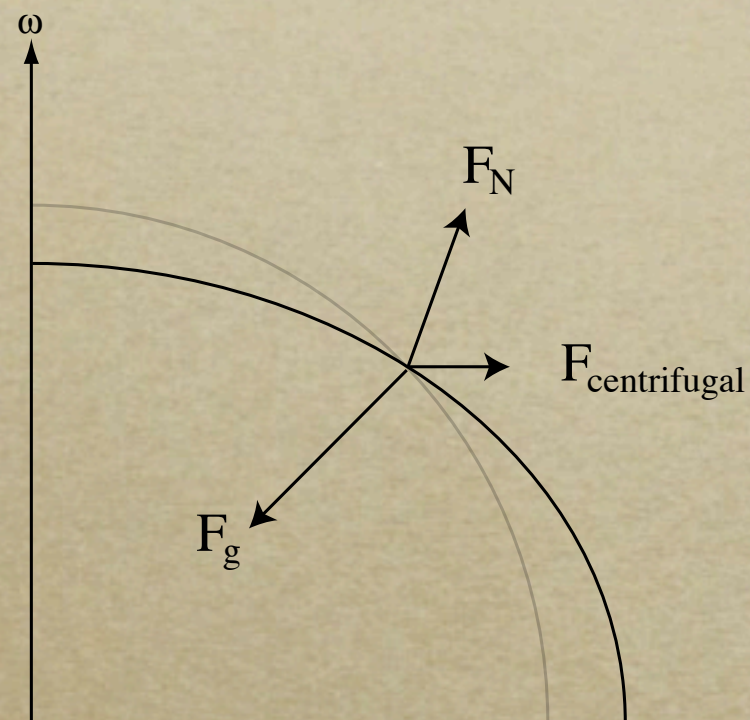
Coordinate transformation

$$\mathbf{a}_f = \mathbf{a}_r + 2\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Equations of motion in inertial coordinates

$$\mathbf{a}_f = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Inertial Oscillations



Durran shows that a component of gravity is present in the inertial frame.

Inertial Oscillations

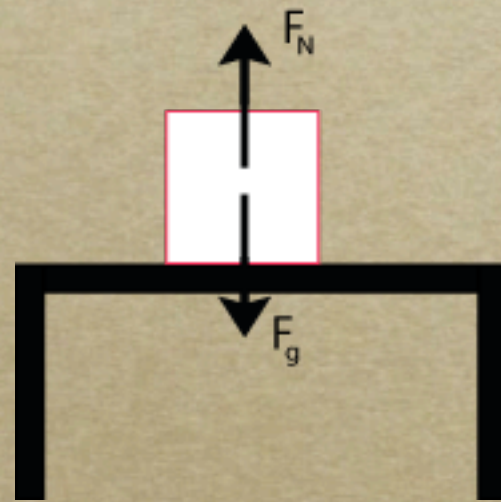
Because there's a force in the inertial frame, Durran concludes that the oscillations aren't inertial. He suggests renaming them.

This implies that the accelerometer on our float does NOT read zero.

Inertial Oscillations

The Experiment

Place an accelerometer on a table. What does it read?



$$\mathbf{a}_f = \mathbf{F}_N + \mathbf{F}_g$$

Inertial Oscillations

The Experiment

Place an accelerometer on a table. What does it read?

The Answer

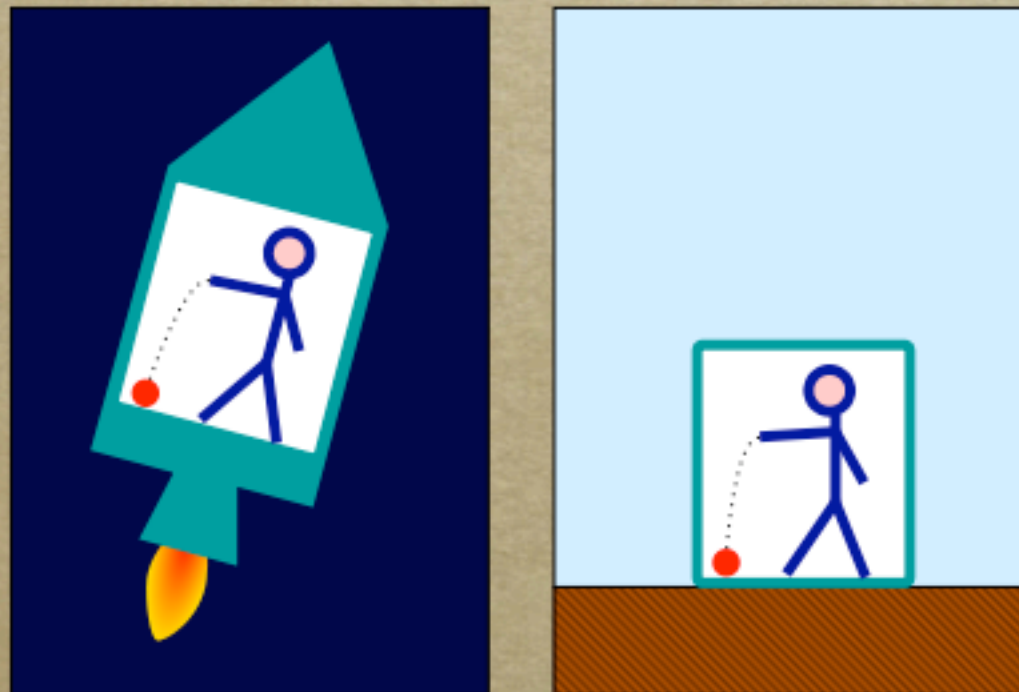
$$+9.8 \frac{m}{s^2}$$

The accelerometer only measures the normal force, not gravity

Inertial Oscillations

Einstein Elevator Experiment

It is impossible to distinguish between an accelerating elevator and an elevator at rest in a gravitational field.



Inertial Oscillations

Instead of writing,

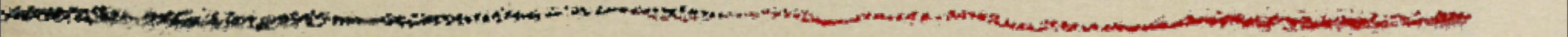
$$\mathbf{a}_f = \mathbf{F}_N + \mathbf{F}_g$$

We should have written,

$$\mathbf{a}_f - \mathbf{F}_g = \mathbf{F}_N$$

When Brian Greene says that gravity is the ‘fabric of the cosmos’, this is part of what he means. Gravity is a local description of the geometry and enters the equations like other metric terms.

Inertial Oscillations



The component of gravity Durran found to be forcing the inertial oscillation isn't measurable.

When the float's accelerometer reads zero, the float was in an inertial oscillation.

Inertial Oscillations

- *The accelerometer reads zero.*
- *Inertial oscillations really are inertial!*
- *Gravity is not a force like pressure, etc.*
- *Durran created a false semantic dichotomy. No experiment can distinguish motion in an inertial oscillation from an inertial frame.*

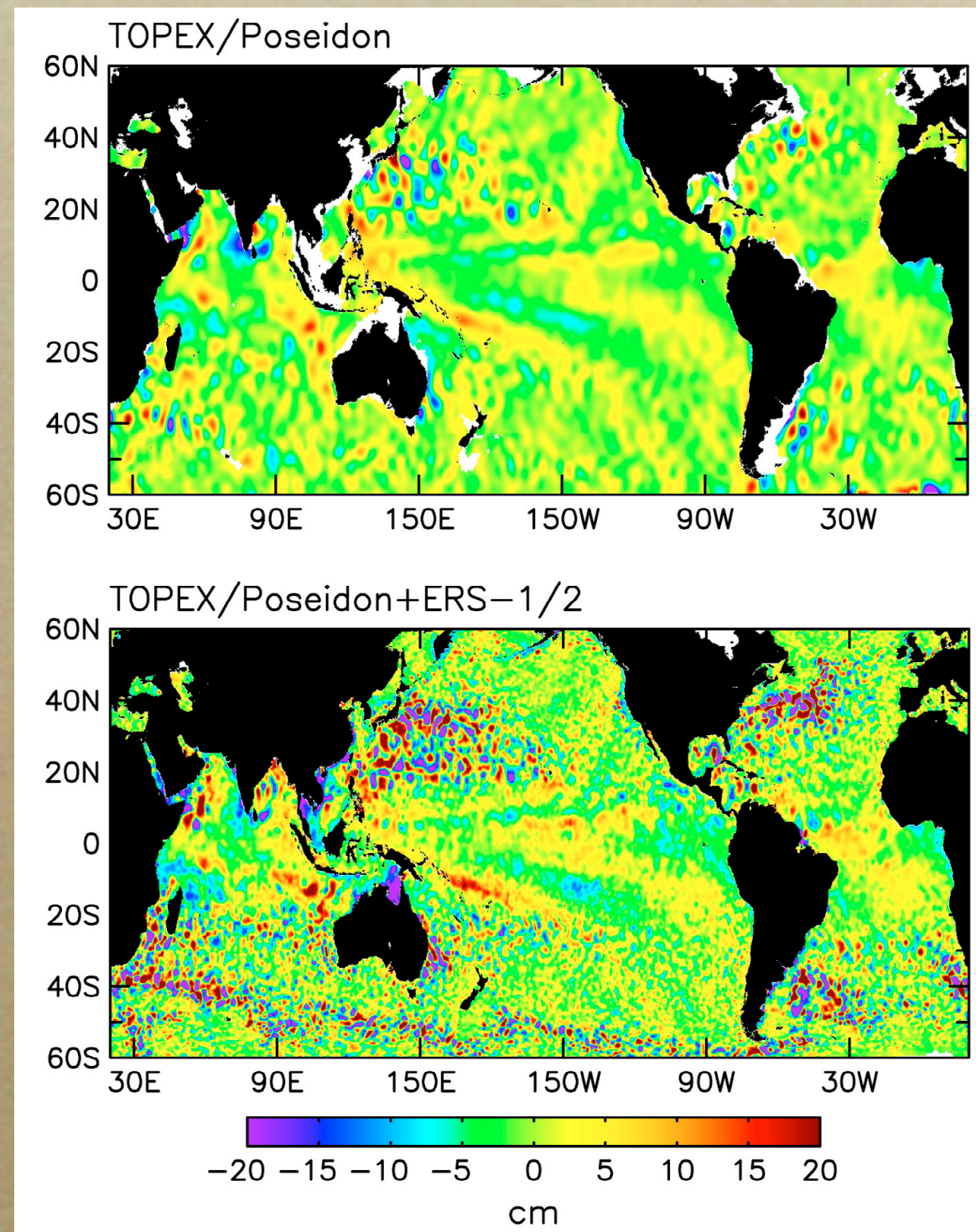
Inertial Oscillations

- *Exact solutions can be found.*
- *Slow westward drift, $U = -\frac{c^2 \beta_0}{f_0^2}$*
- *Frequency is subinertial.*
- *Other fun stuff...*

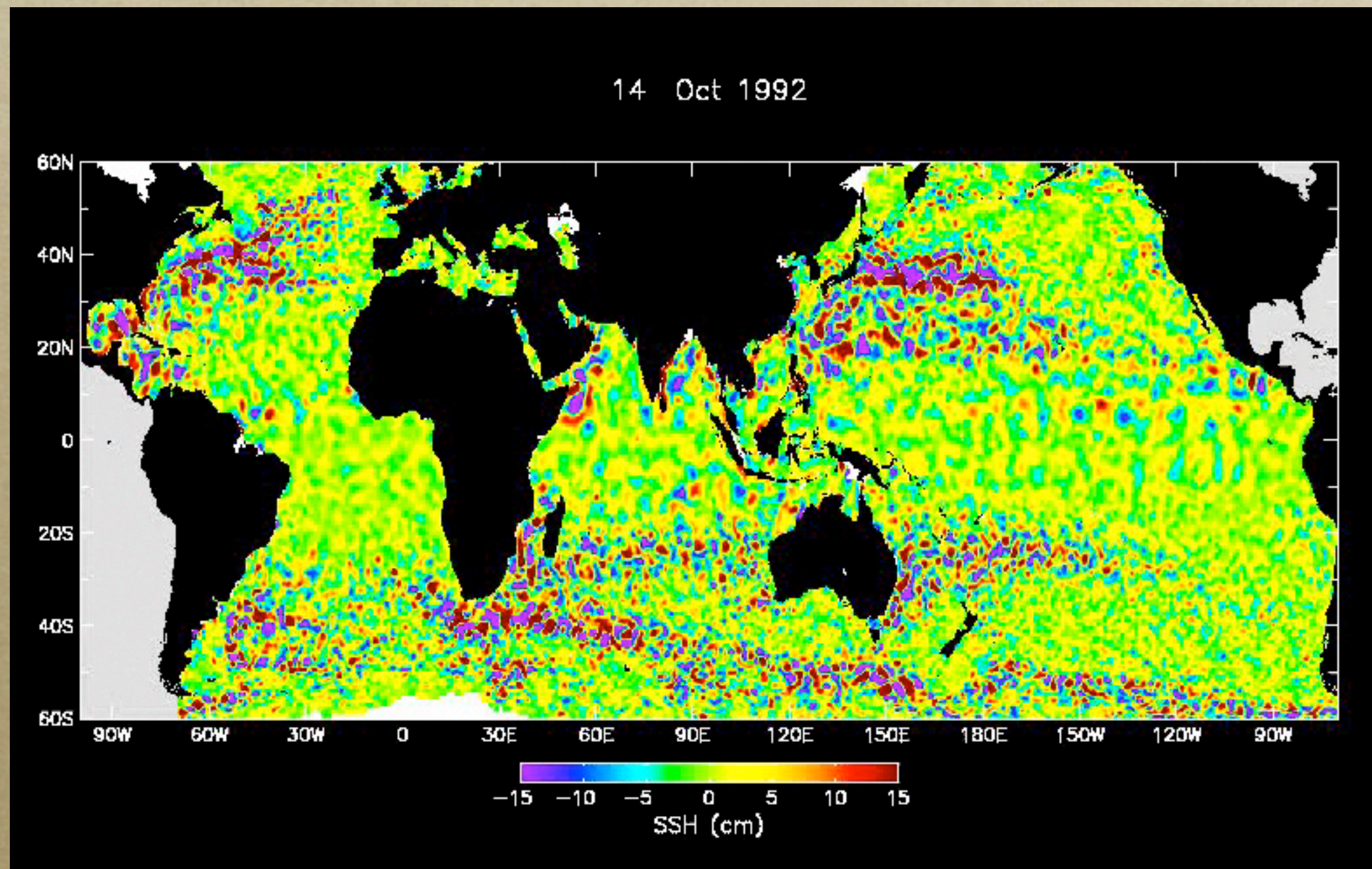
Disk on the Earth

- *Analogy with a vortex?*
- *Also has slow westward drift.*
- *Forcing is identical to a spinning top.*

Quasigeostrophic Eddies



Quasigeostrophic Eddies



Quasigeostrophic Eddies

From Observations, Chelton 2011

- *Typical lifetimes*
- *Typical scales (length, height, speed)*
- *Meridional deflection*
- *Propagation speed*
- *Spectral characteristics*

Quasigeostrophic Eddies

Quasigeostrophic Potential Vorticity Equation

$$\frac{\partial}{\partial t} (\nabla^2 \eta - \eta) + \frac{\partial \eta}{\partial x} + \beta^{-1} \cdot J(\eta, \nabla^2 \eta) = 0$$

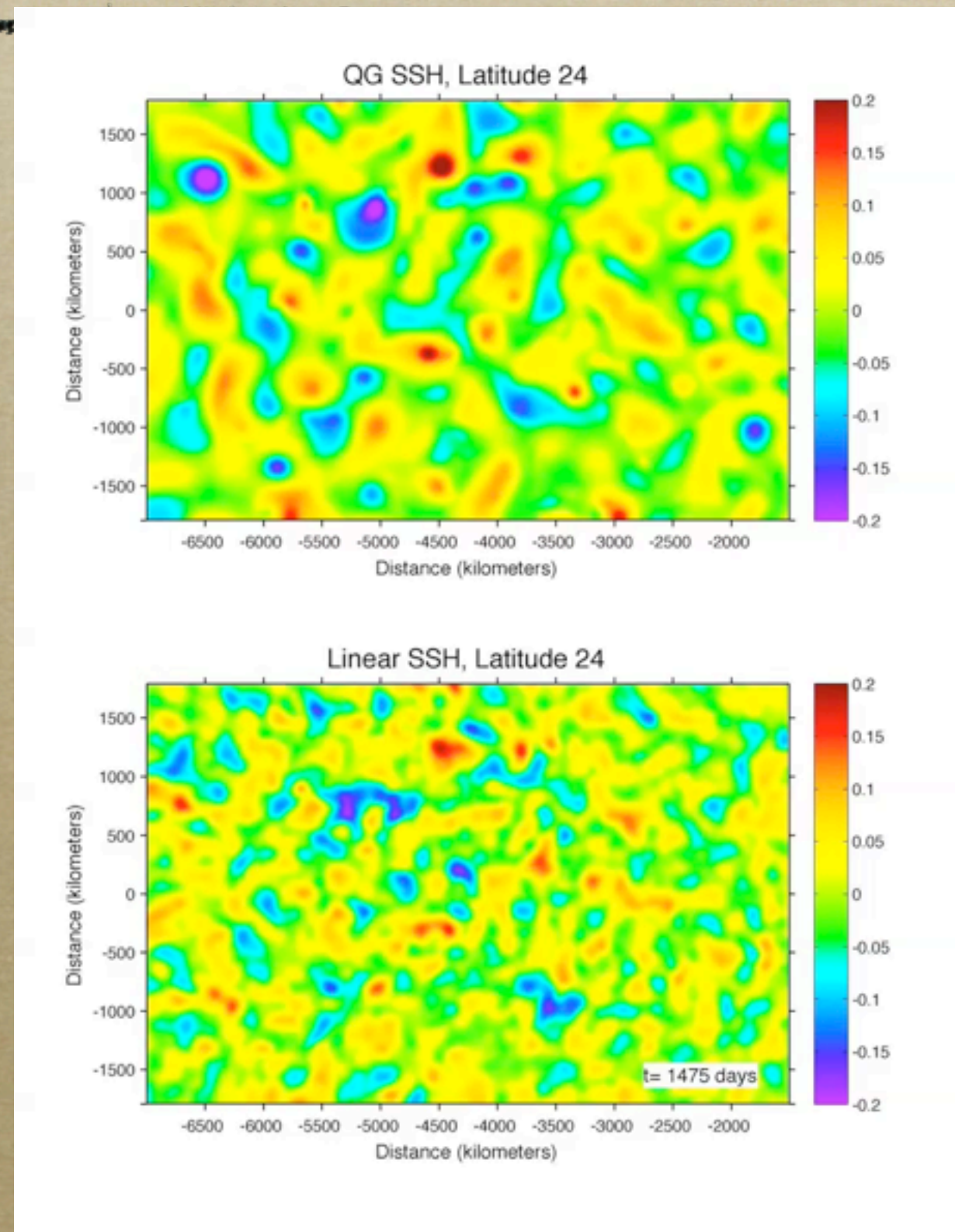
where

$$J(a, b) = a_x b_y - a_y b_x$$

and

$$\beta^{-1} = O(100)$$

Quasigeostrophic Eddies



Quasigeostrophic Eddies

Does the QG model match the observations?

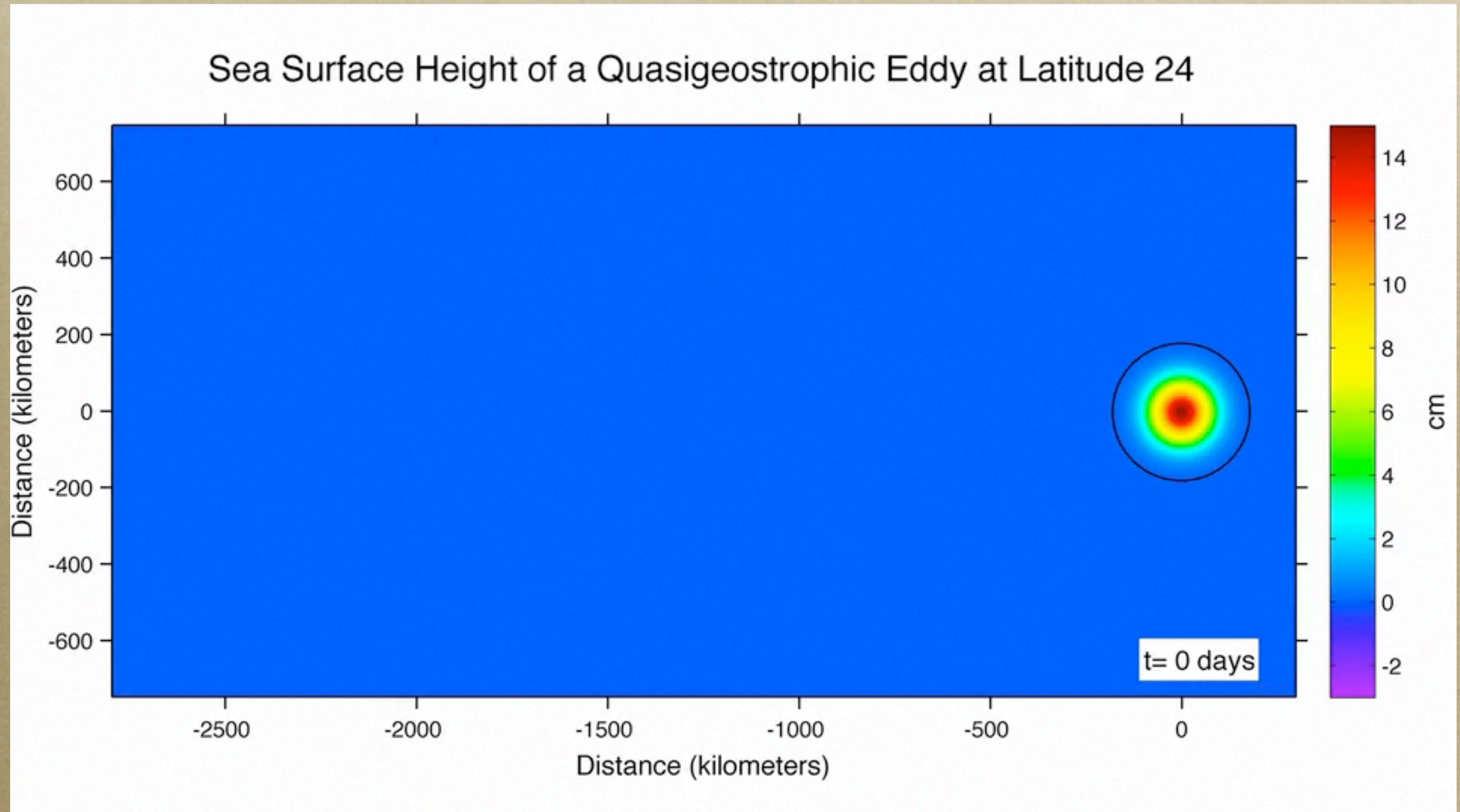
- *Spectra matches really well.*
- *Meridional deflection is correct (to lowest order).*
- *Propagation speeds match extremely well.*

Quasigeostrophic Eddies

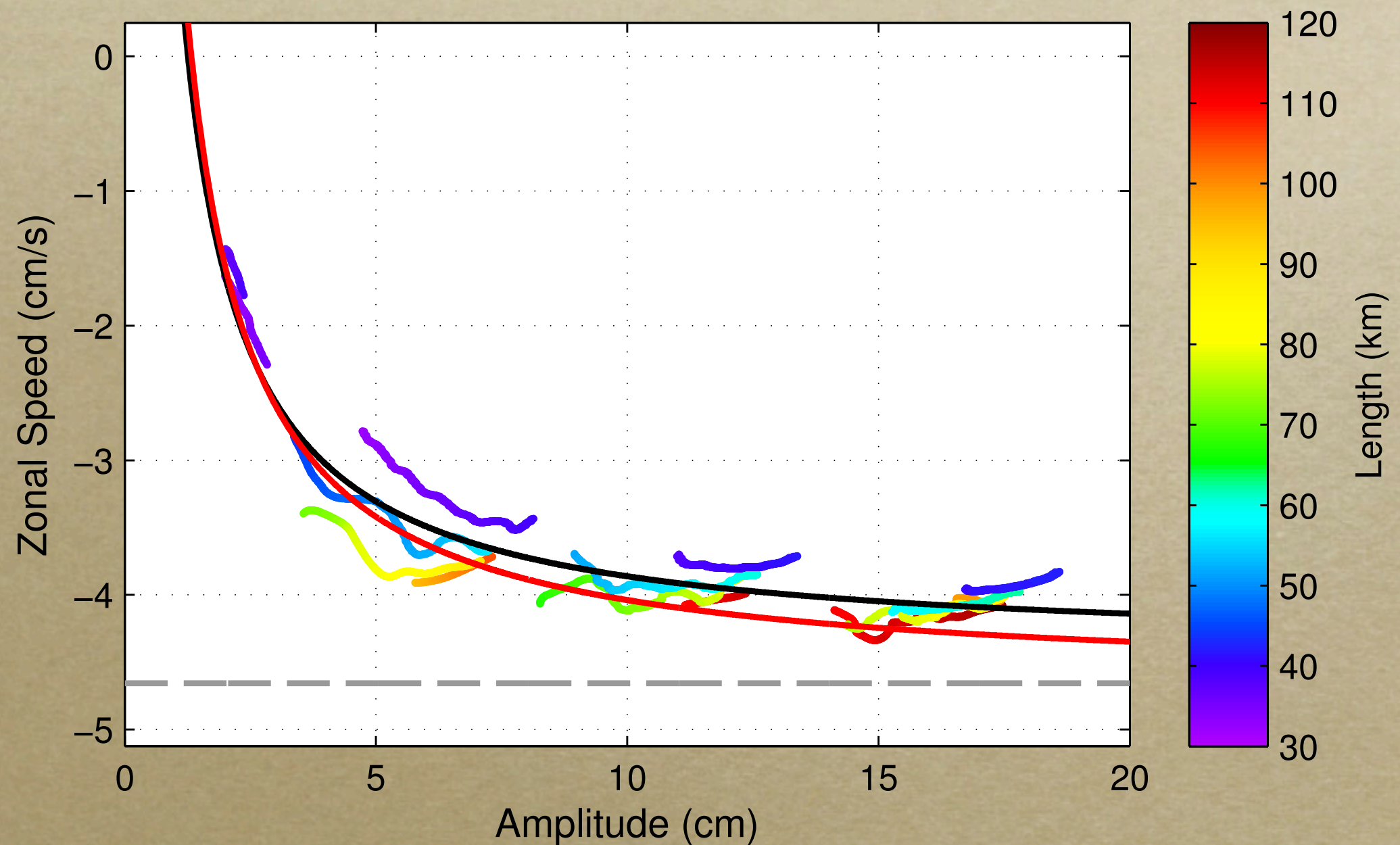
Monopole Study

- *How does a Gaussian eddy evolve?*
 - *Shape*
 - *Fluid transport properties*
 - *Zonal and meridional speeds*
 - *Amplitude and length decay rates*
 - *Potential vorticity and energy*

Quasigeostrophic Eddies

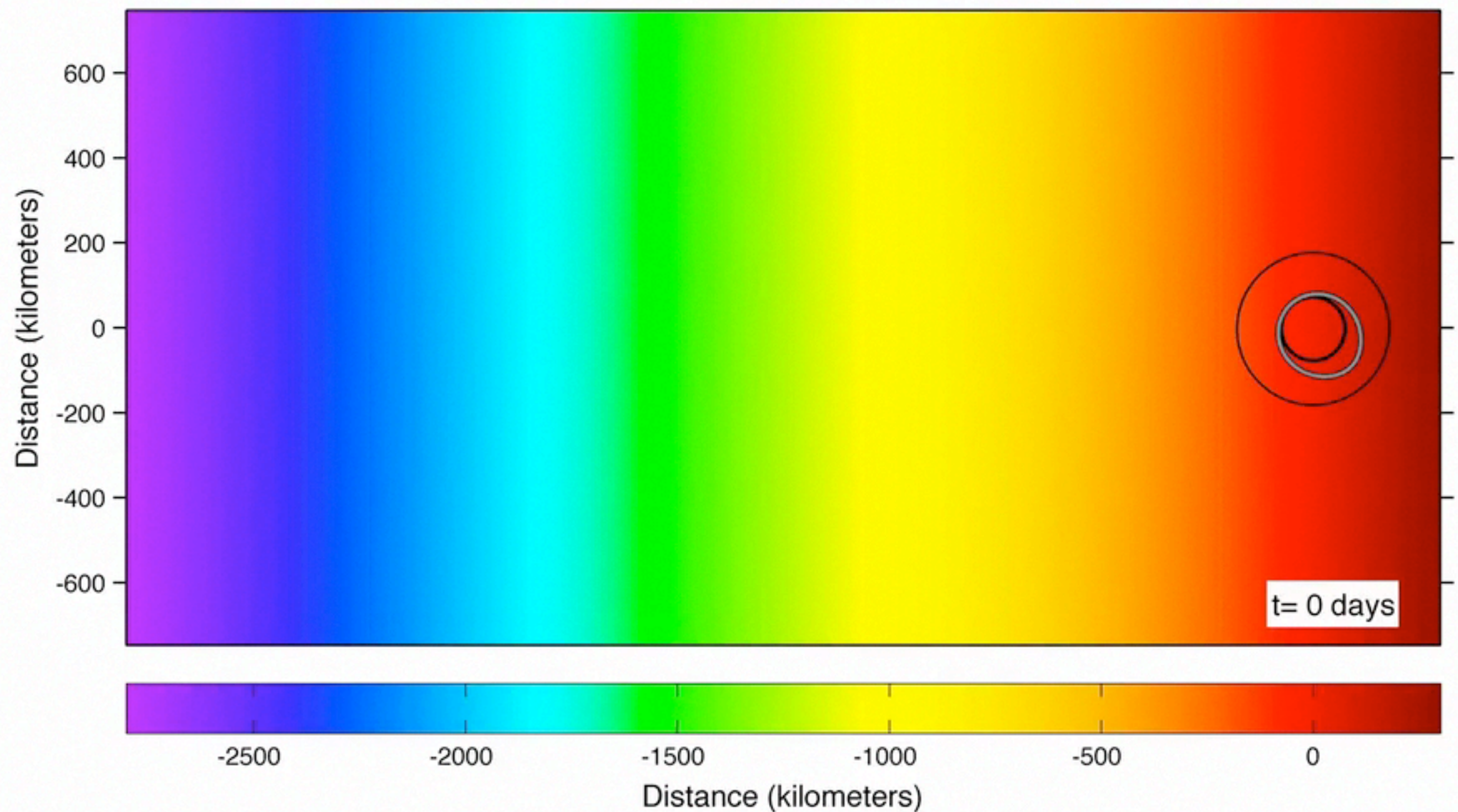


Quasigeostrophic Eddies

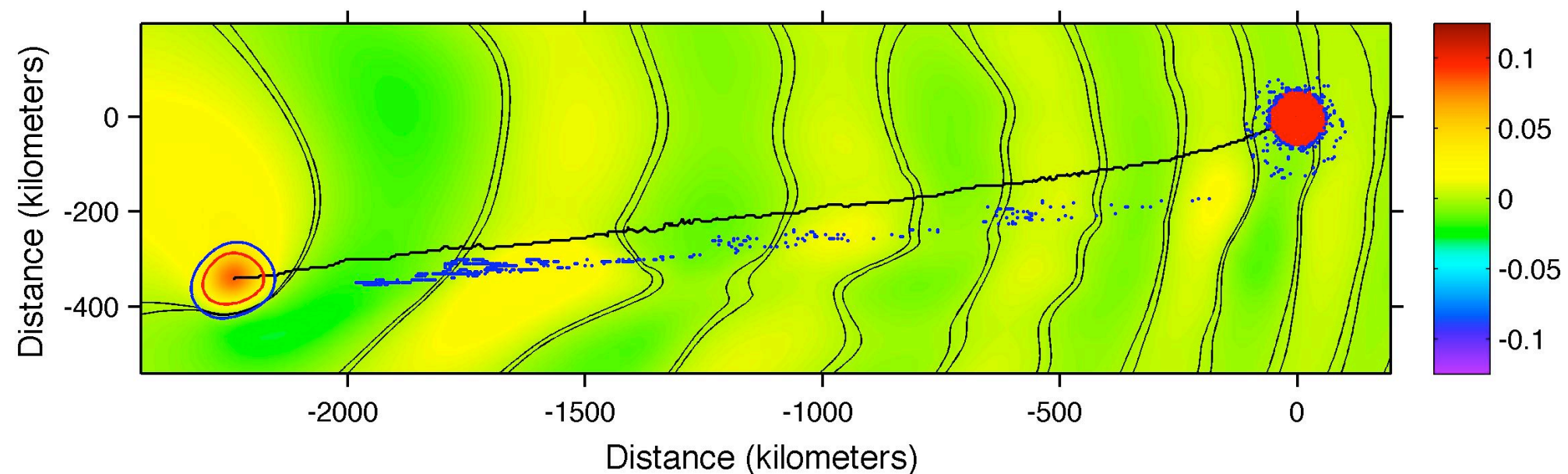


Quasigeostrophic Eddies

Passive Tracer Advected by a Quasigeostrophic Eddy at Latitude 24



Quasigeostrophic Eddies



Eddy core (region within RV zero contour) never entrains new fluid, only loses original fluid.

Eddy ring (region between trapped fluid contour and the core) continuously acquires and deposits fluid.

Quasigeostrophic Eddies

- *Eddy core and ring carry fluid very differently.*
- *Propagation speeds governed primarily by amplitude, and to a lesser extent length scale.*
- *Eddy reaches quasistable state after ~200 days.*
- *QG eddies closely match observations.*