

Theoretical Approaches to Understanding Satellite Altimetry Tracked Eddies

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Oregon State University

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Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Thesis Overview

Mathematical Approaches to the Physics of Mesoscale Oceanography

Advisor: Roger Samelson

1. Introduction
2. Wave versus eddy interpretation of satellite altimetry observations; long-term evolution of quasi-geostrophic eddies
 - ▶ In collaboration with Dudley Chelton
3. Near-geostrophic approximations to the spherical shallow water equations
4. Group foliation of equations in geophysical fluid dynamics
 - ▶ In collaboration with Juha Pohjanpelto
5. The forces of inertial oscillations
6. Conclusions

**Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies**

Observations

Wave Interpretation
Eddy Interpretation
Attributes

**Linear vs
Nonlinear**

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Talk Overview

Theoretical Approaches to Understanding Satellite Altimetry Tracked Eddies

1. Review satellite altimetry observations: wave and eddy interpretations.
2. Show that linear Rossby waves fail to explain observations, nonlinear quasi-geostrophy (QG) is required.
3. Describe the properties of quasi-geostrophic eddies.
4. Show that QG fails to explain propagation speed.
5. Argue that the Flierl-Petviashvili (FP) equation and extensions may make appropriate corrections.

Theoretical Approaches to Understanding Satellite Altimetry Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

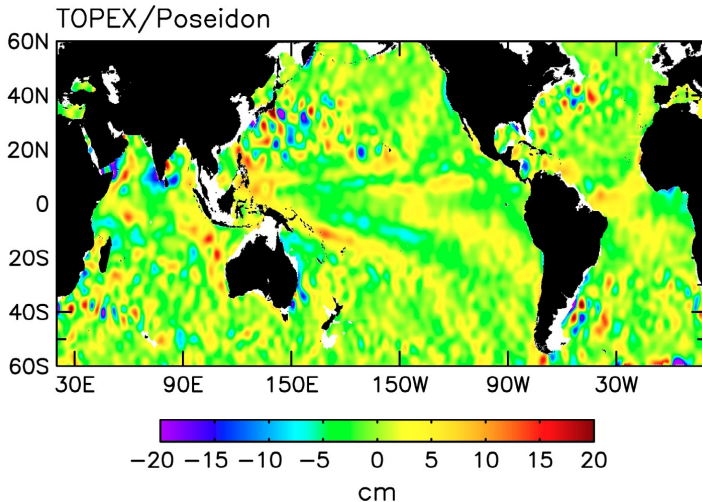
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Global Observations of Oceanic Rossby Waves

[Chelton & Schlax, 1996]



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation

Eddy Interpretation

Attributes

Linear vs
Nonlinear

Numerical Setup

Monopole

Eddy Seeding

QG Monopoles

Shape

Trapped Fluid

Zonal & Meridional
Speed

Mini Conclusions

FP Equation

The FP Monopole

Balance Equations

Scales

Extended Theories

Mini Conclusions

Conclusions

Rossby Wave Interpretation

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

- Vertical structure: assumed first baroclinic mode.

Linear Quasi-Geostrophic Potential Vorticity Equation

$$\frac{\partial}{\partial t} (\nabla^2 \eta - \eta) + \frac{\partial \eta}{\partial x} = 0,$$

is solved by linear combinations of

$$\eta(x, y, t) = N_0 \cos(kx + ly - \omega t + \phi)$$

where

$$\omega = \frac{-k}{k^2 + l^2 + 1}.$$

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

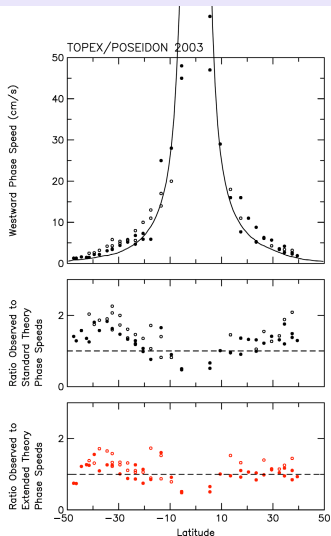
Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Long Wave Phase Speed



- ▶ Feature resolution limited to ~ 200 km.
- ▶ Observed phase speed faster than the long wave limit.
- ▶ Result spurred new theories for faster phase speeds: mean flow effect, ocean-atmosphere interaction, topography effect, etc.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Global Observations of Large Oceanic Eddies

[Chelton et. al., 2007]

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs Nonlinear

Numerical Setup
Monopole
Eddy Seeding

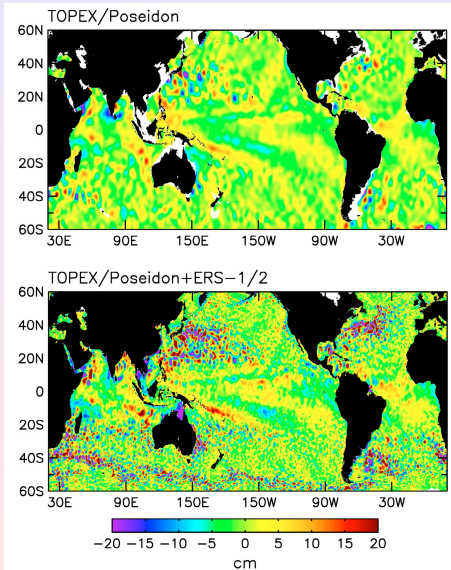
QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

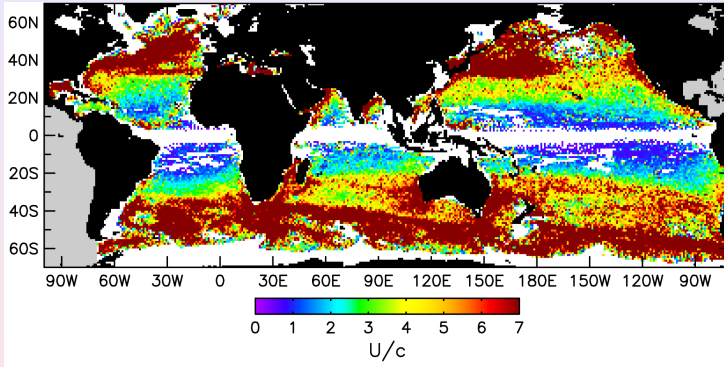
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions



Ratio of Fluid Speed (U) to Propagation Speed (c)



- ▶ $U > c$ suggests advection is important.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Eddy Interpretation

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Enhanced resolution $\implies$ new interpretation

- ▶ Feature resolution increased to ~ 100 km.
- ▶ Observations now appear to be dominated by eddies.
- ▶ Need to include the advection of relative vorticity.

Quasi-Geostrophic Potential Vorticity Equation

$$\frac{\partial}{\partial t} (\nabla^2 \eta - \eta) + \frac{\partial \eta}{\partial x} + \beta^{-1} \cdot J(\eta, \nabla^2 \eta) = 0,$$

where

$$J(a, b) = a_x b_y - a_y b_x.$$

and $\beta^{-1} = O(100)$.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes

Focus on three observational attributes:

- Coherence. Structures trackable for long time periods.

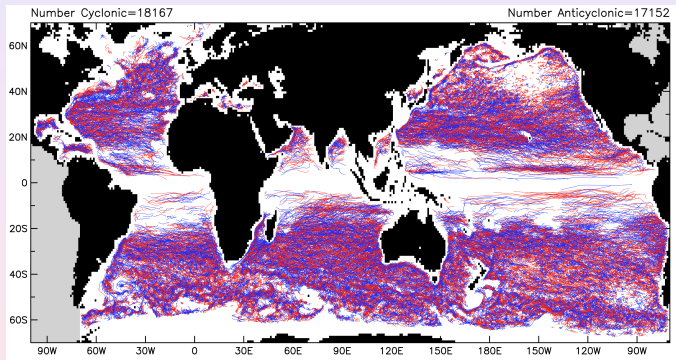


Figure: ~35,000 cyclonic and anti-cyclonic eddies with lifetimes greater than 16 weeks

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes

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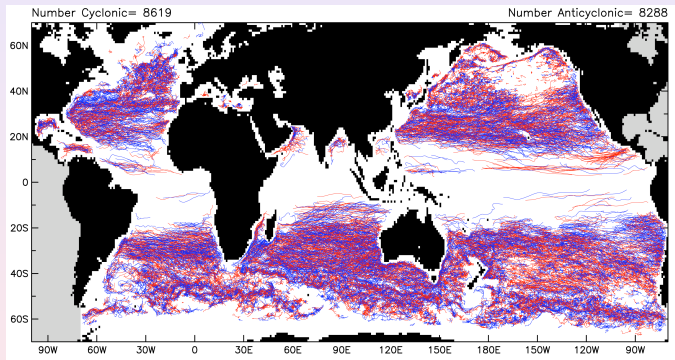


Figure: ~17,000 cyclonic and anti-cyclonic eddies with lifetimes greater than 6 months

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes

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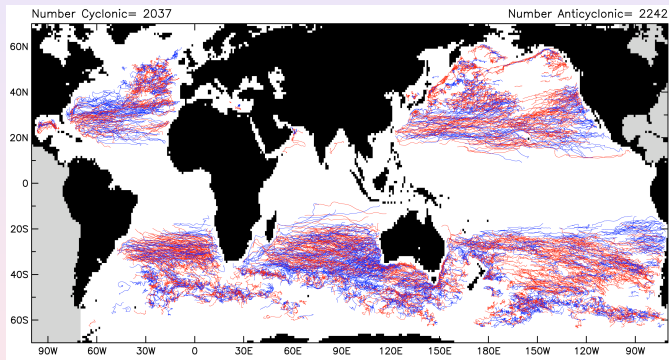


Figure: ~4,000 cyclonic and anti-cyclonic eddies with lifetimes greater than 1 year

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

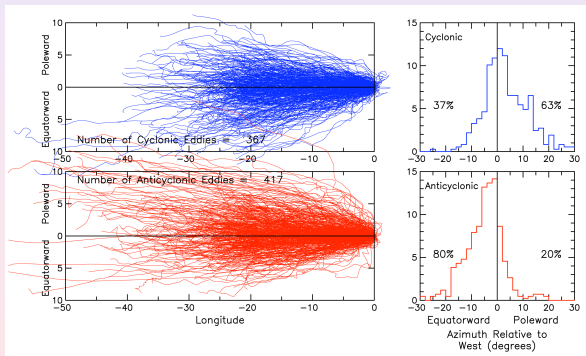
The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes

Focus on three observational attributes:

- Coherence. Structures trackable for long time periods.
- Meridional deflection. Cyclonic eddies deflect poleward, anti-cyclonic deflect equatorward.



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

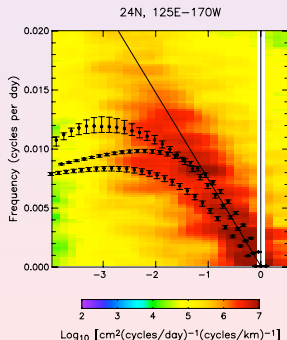
The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes

Focus on three observational attributes:

- ▶ Coherence. Structures trackable for long time periods.
- ▶ Meridional deflection. Cyclonic eddies deflect poleward, anti-cyclonic deflect equatorward.
- ▶ Spectral properties. Wavenumber-frequency spectra appear non-dispersive.



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations
Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear
Numerical Setup
Monopole
Eddy Seeding

QG Monopoles
Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation
The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes

Focus on three observational attributes:

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- ▶ Spectral properties. Wavenumber-frequency spectra appear non-dispersive.

Can we explain any of this with linearized QG theory?

If not, how well does non-linear QG do?

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations
Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear
Numerical Setup
Monopole
Eddy Seeding

QG Monopoles
Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation
The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Numerical Setup

- ▶ Pseudo-spectral with Runge-Kutta time stepping.
- ▶ Sponge-layers on all the boundaries.
- ▶ Damping with spectral vanishing viscosity.

Modeled Equation

$$\frac{\partial}{\partial t} (\nabla^2 \eta - \eta) + \frac{\partial \eta}{\partial x} + \beta^{-1} \cdot J(\eta, \nabla^2 \eta) = 0,$$

where

$$\beta^{-1} = \begin{cases} 0 & \text{linearized ,} \\ 60 & \text{non-linear.} \end{cases}$$

at latitude 24 where $L_R = 47$ km, $c_k = 4.7$ cm/s and $(\beta_0 L_R)^{-1} = 12$ days.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Linear Rossby Wave Dynamics

(Linear Rossby Wave Dynamics)

Nonlinear Quasigeostrophic Dynamics

(Nonlinear Quasigeostrophic Dynamics)

**Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies**

Observations

Wave Interpretation
Eddy Interpretation
Attributes

**Linear vs
Nonlinear**

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes Revisited

Coherence?

- ▶ Linear model shows quick dispersion. Amplitude reduced to 10% of original after 150 days.
- ▶ Nonlinear QG model shows strong coherence. Amplitude still 75% of original after 365 days.

Meridional Deflection?

- ▶ Linear model shows no meridional deflection.
- ▶ QG model shows equatorward deflection for the anticyclonic eddy.

Symmetry of the Quasigeostrophic Potential Vorticity Equation

If $\eta(x, y, t)$ is a solution, then so is $\tilde{\eta} = -\eta(x, -y, t)$. This implies that what is true of anticyclonic eddies is also true of cyclonic eddies if you flip equatorward and poleward.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Eddy Seeding

(Nonlinear Quasigeostrophic Dynamics)

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

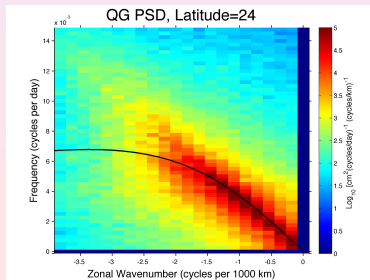
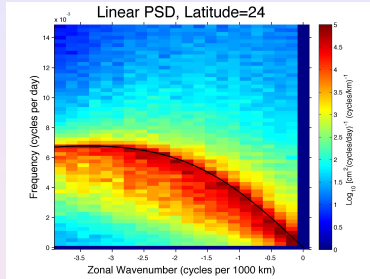
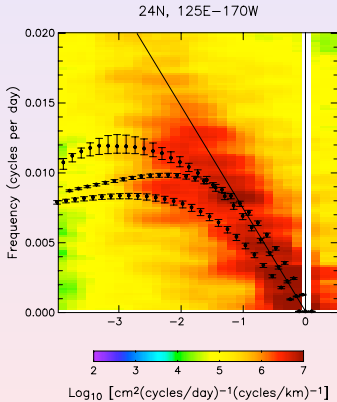
Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Eddy Seeding



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

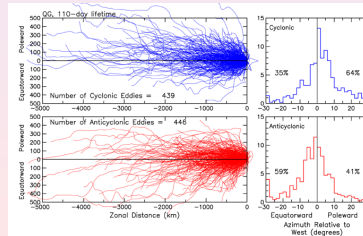
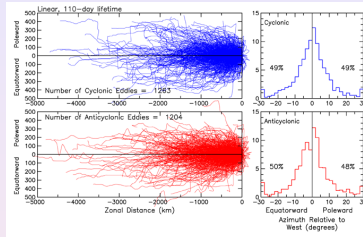
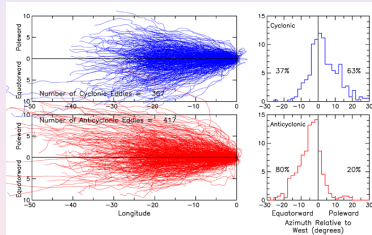
Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Eddy Seeding



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes Revisited

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Spectra?

- ▶ Nonlinear model is a far better match than the linear model.

Meridional Deflection?

- ▶ Linear model shows equal preference for meridional deflection, contrary to the observations.

The satellite altimetry signal more closely matches nonlinear quasi-geostrophic theory than linear Rossby wave theory.

- ▶ The observations show quasi-geostrophic eddies, and not linear Rossby waves.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Attributes Revisited

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

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But how well does quasi-geostrophy actually do?

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Quasi-geostrophic Monopoles

Primary Objective

Establish basic properties of evolution for quasi-geostrophic monopoles to compare to the observations and other theories.

- ▶ Shape.
- ▶ Fluid transport properties.
- ▶ Zonal and meridional speeds.
- ▶ Amplitude and length decay rates.
- ▶ Potential vorticity and energy.

(Nonlinear Quasigeostrophic Dynamics)

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Experimental Design

- ▶ Initialized with (5, 10, 15, 20) cm amplitudes and (40, 60, 80, 100, 120) km length scales.
- ▶ Allowed to evolve for 730 days.
- ▶ Nearly all evolved into quasi-stable state.
- ▶ Non-gaussian shapes also evolved into quasi-stable state.
- ▶ Focus on quasi-stable state.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Historical Background: β -gyre

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Formation of the β -gyre

Initial gaussian shape evolves to asymmetric shape within the first 12 days.

- ▶ First described by [Fiorino and Elsberry, 1989].
- ▶ Analytical prediction does pretty well [Sutyrin and Flierl, 1994], [Reznik and Dewar, 1994]...
- ▶ ... until Rossby wave radiation dominates the evolution.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Historical Background: 'Quasi-Stable' State

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

'Quasi-Stable' State Hypothesis

After 12 days the monopole tends quickly towards a quasi-stable slowly decaying state.

[McWilliams & Flierl, 1979]

- ▶ Meridional & zonal propagation speeds then asymptote towards length dependent phase speeds (60-80 days).
- ▶ Amplitude decay determined by damping.

[Korotaev, 1997]

- ▶ Amplitude decay actually determined by Rossby wave radiation [Flierl, 1984].
- ▶ Analytical prediction for speed off by 30% after about 80 days.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

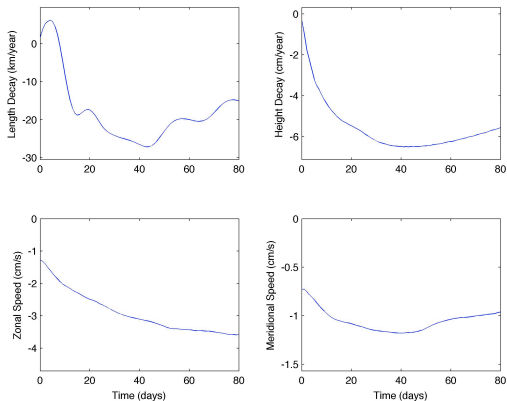
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Historical Background: 'Quasi-Stable' State

- Do changes in the eddy's properties stabilize around 80 days?



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

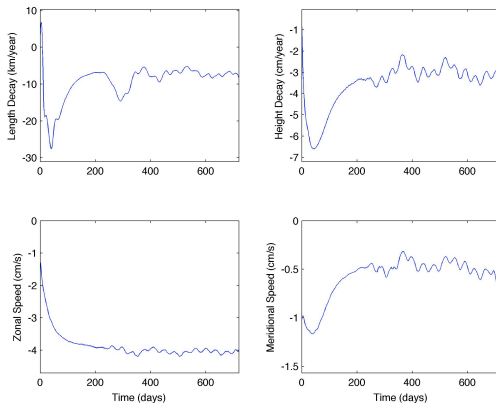
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Historical Background: 'Quasi-Stable' State

► Hmmm, maybe not...



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

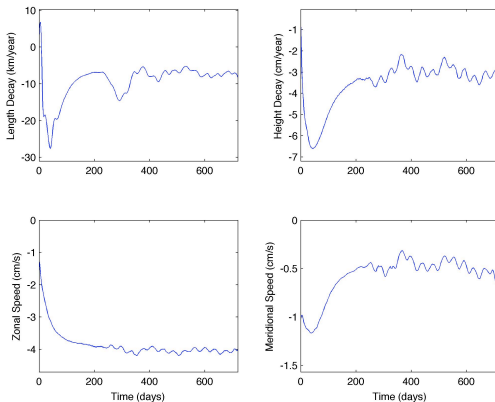
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Monopole Evolution: Three States

1. Formation of the β -gyre during the first 10 days.
2. Adjustment period up to 200 days.
3. Quasi-stable slowly decaying state after 200 days.



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

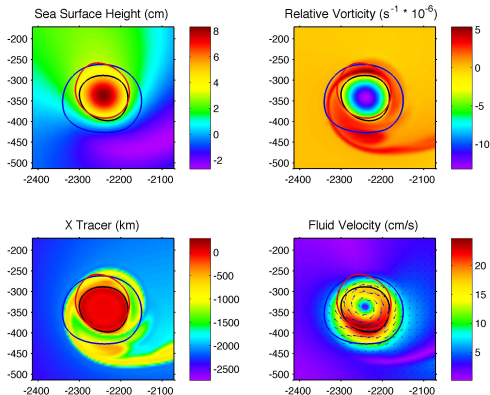
The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Shape Properties

Three contours:

1. E-folding height contour
2. Relative vorticity zero contour
3. Trapped fluid contour



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Trapped Fluid: Passive Tracer

(Linear Rossby Wave Tracer Field)

(QG Tracer Field)

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid

Zonal & Meridional
Speed
Mini Conclusions

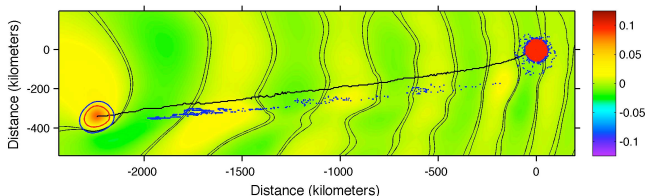
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Trapped Fluid: Floats

- ▶ Eddy path
- ▶ Original location of floats found in the eddy **core** on day 675.
- ▶ Original location of floats found in the eddy **ring** on day 675.



Potential Vorticity Conservation

The floats were also used to diagnose the changes in the contribution of PV conservation.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid

Zonal & Meridional
Speed

Mini Conclusions

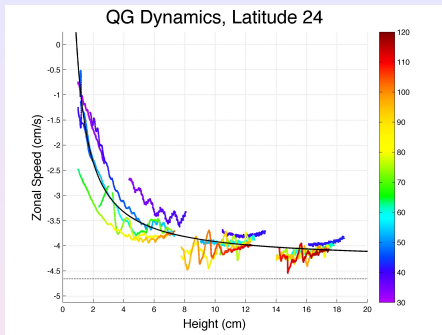
FP Equation

The FP Monopole
Balance Equations
Scales

Extended Theories
Mini Conclusions

Conclusions

Experimental Zonal Speed versus Amplitude



- ▶ $c_x(A) = 4.2A^{-1} - 4.4$ cm/s, suggestive of 4.7 cm/s upper bound.
- ▶ Bigger amplitude, bigger length scale imply faster westward propagation.
- ▶ Relationship not predicted by [McWilliam & Flierl, 1979], [Cushman-Roisin, et. al, 1990], [Korotaev, 1997]? or [Nycander, 2000].

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

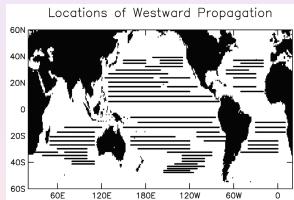
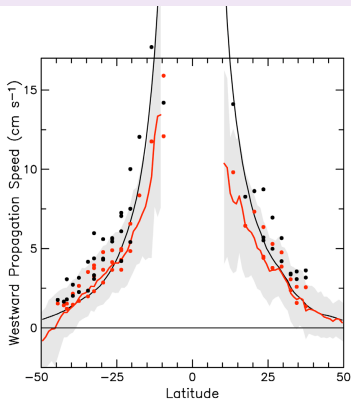
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Observed Zonal Speed versus Latitude

- ▶ —Non-dispersive Rossby wave speed.
- ▶ —Observed average propagation speed.
- ▶ ● Radon speed ('large eddies')
- ▶ ● Tracked eddies.



- ▶ Good: Big features propagate faster than small features.
- ▶ Bad: Observed features overall move too fast.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

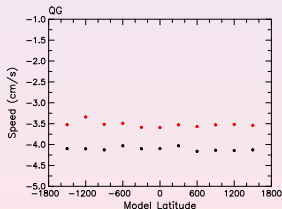
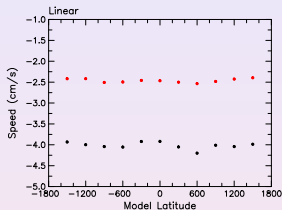
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Model Zonal Speed versus Model Latitude

- ▶ ● Radon speed ('large eddies')
- ▶ ● Tracked eddies.



- ▶ Confirms the radon transform bias towards bigger eddies – and that QG predicts too slow of propagation speeds.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

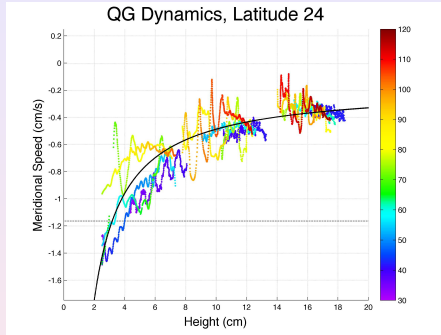
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Experimental Meridional Speed versus Amplitude

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies



- ▶ $c_y(A) = -3.7A^{-1} - 0.13 \text{ cm/s}$.
- ▶ Smaller amplitude, smaller length scale imply faster meridional propagation.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid

Zonal & Meridional
Speed

Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales

Extended Theories
Mini Conclusions

Conclusions

Mini Conclusions

- ▶ There are **three** states during monopole evolution: β -gyre formation, adjustment and quasi-stable.
- ▶ Eddy core (region within RV zero contour) never entrains new fluid, only loses original fluid.
- ▶ Eddy ring (region between trapped fluid contour and the core) continuously acquires and deposits fluid.
- ▶ Eddy propagation rates governed by amplitude & length, tentatively matching observations.
- ▶ BUT, propagation rates are **always** slower than the non-dispersive Rossby wave speed.
- ▶ Quasi-geostrophy cannot explain the observed propagation speeds without corrections. (mean-flow?, amplitude?)

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed

Mini Conclusions

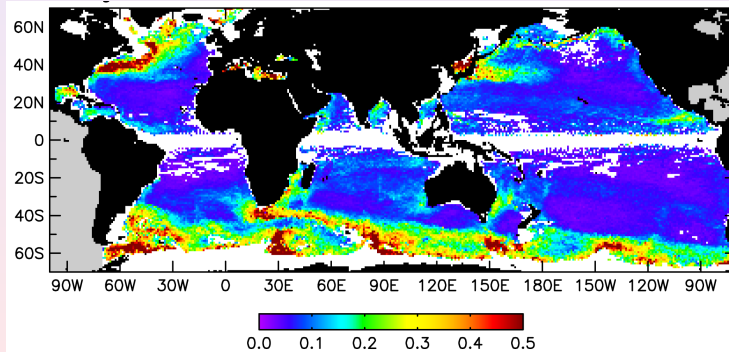
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

One Source of Nonlinearity

Ratio of observed surface displacement to equivalent depth is not small. This suggests that next to leading order terms matter.



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

An Extended Theory

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Flierl-Petviashvili Equation

$$\frac{\partial}{\partial t} \left(\nabla^2 \eta - \left(\eta - \frac{1}{2} \eta^2 \right) \right) + \eta_x + \beta^{-1} J(\eta, \nabla^2 \eta) = 0$$

- ▶ Addition is the η^2 term.
- ▶ Flierl also looked for eddy like structures, but a shear flow was his primary source of nonlinearity.
- ▶ Petviashvili considered the Great Red Spot on Jupiter.
- ▶ Charney & Flierl later showed results from height non-linearity.
- ▶ de Szoeke showed that this equation also results from a thermobaric effect.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

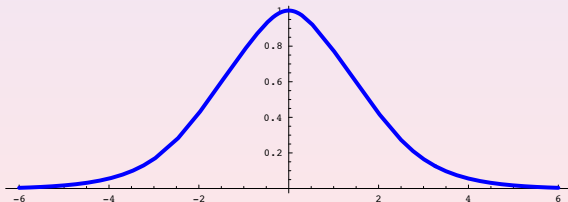
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

The FP Monopole

- ▶ One known radial solution, found numerically.
- ▶ Does provide a speedup from linear theory.
- ▶ Positive sea surface height anomaly only (no cyclonic solutions).
- ▶ Amplitude inversely proportional to the square of the length.



Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

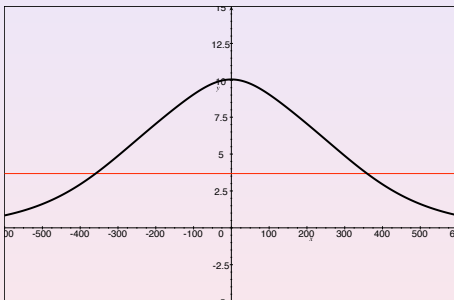
FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Is the FP monopole a good solution?

- ▶ At latitude 24, 10 cm amplitude requires e-folding diameter of 725 km! Equation & solution not valid.



- ▶ At latitude 55, 10 cm amplitude requires e-folding diameter of 350 km. Maybe okay.

Where is the FP monopole a good solution?

- ▶ Too big of an amplitude, we need the next order height term.
- ▶ Too wide, we need spherical geometry.
- ▶ Too steep (large U), we other next order terms.

In order to answer this we developed a technique to

1. evaluate the relative importance of missing terms and
2. estimate which terms to a theory are valid at given scales.

Terminology

For some scales we can reduce the primitive equations to *balance dynamics* and *balance relations*.

Balance Dynamics

Describe the evolution of the system, such as the QGPVE:

$$\frac{\partial}{\partial t} (\nabla^2 \eta - \eta) + \frac{\partial \eta}{\partial x} + \beta^{-1} \cdot J(\eta, \nabla^2 \eta) = 0,$$

Balance Relations

Describe the evolution of the other variables

$$v = \eta_x, \quad u = -\eta_y.$$

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Review of a Typical Asymptotic Derivation [Pedlosky, 1987]

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

1. Choose non-dimensionalized 'primitive' equations

$$v = \eta_x + \epsilon u_t$$

$$u = -\eta_y - \epsilon v_t$$

$$\epsilon \eta_t + \epsilon(u\eta_x + v\eta_y) = -(1 + \epsilon\eta)(u_x + v_y).$$

2. Assume $\epsilon \ll 1$ and expand u , v , η as perturbation series,

$$v = v^0 + \epsilon v^1 + \epsilon^2 v^2 + O(\epsilon^3)$$

3. Insert into primitive equations & require each order vanish,
4. Take the curl at each order of ϵ ,
5. Shuffle around to get potential vorticity equation in terms of η^0 .

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

An Iterated Approach (similar to [Allen, 1993])

1. Choose non-dimensionalized 'primitive' equations, e.g.

$$v = \eta_x + \epsilon u_t$$

$$u = -\eta_y - \epsilon v_t$$

2. Find the balance relations by substituting u equation into v equation and vice versa,

$$v = \eta_x - \epsilon \eta_{yt} + \epsilon^2 v_{tt}$$

$$u = -\eta_y - \epsilon \eta_{xt} - \epsilon^2 u_{tt}$$

3. Find the balance dynamics from potential vorticity conservation, e.g.,

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \left[f + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \eta \right]$$

Method

1. Start with the shallow water equations in spherical coordinates.
2. Non-dimensionalize the equations and you find,

$$\epsilon_T = \frac{1}{2\omega T}, \epsilon_R = \frac{U}{2\omega L}, \epsilon_\beta = \frac{U}{2\omega R}, G = \frac{gN_0}{2\omega UL}, \epsilon_H = \frac{N_0}{D}$$

3. Don't fix the parameters!
4. Insert u , v equations into the potential vorticity equation twice.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

The Complete Equation: Don't Panic!

$$\begin{aligned}
 0 = & \epsilon_{\beta} G \cdot \frac{\cos \theta}{\sin \theta} \frac{\eta_{\bar{x}}}{\cos \theta} \\
 & + \epsilon_T \epsilon_R G \cdot \frac{1}{\sin \theta} \nabla^2 \eta_{\bar{t}} \\
 & - \epsilon_T \epsilon_{\beta} G \cdot \frac{1 + \cos^2 \theta}{\sin^2 \theta \cos \theta} \eta_{\bar{t}\bar{y}} \\
 & + \epsilon_R^2 G^2 \cdot \frac{1}{\sin^2 \theta} \frac{1}{\cos \theta} \left[-\frac{\eta_{\bar{y}} \eta_{\bar{x}\bar{x}\bar{x}}}{\cos^2 \theta} + \eta_{\bar{x}} \eta_{\bar{y}\bar{y}\bar{y}} - \eta_{\bar{y}} \eta_{\bar{x}\bar{y}\bar{y}} + \frac{\eta_{\bar{x}} \eta_{\bar{x}\bar{x}\bar{y}}}{\cos^2 \theta} \right] \\
 & + \epsilon_R \epsilon_{\beta} G^2 \cdot \frac{\cos \theta}{\sin^3 \theta} \left[-3 \frac{\eta_{\bar{x}} \eta_{\bar{y}\bar{y}}}{\cos \theta} + \frac{1 + \cos^2 \theta}{\cos^2 \theta} \frac{\eta_{\bar{y}} \eta_{\bar{x}\bar{y}}}{\cos \theta} + \frac{2 - 3 \cos^2 \theta}{\cos^2 \theta} \frac{\eta_{\bar{x}} \eta_{\bar{x}\bar{x}}}{\cos^3 \theta} \right] \\
 & - \epsilon_R \epsilon_{\beta} G \cdot \frac{\eta_{\bar{y}\bar{y}}}{\cos \theta} \\
 & + \epsilon_{\beta}^2 G^2 \cdot \frac{\cos^4 \theta + 3 \cos^2 \theta - 1}{\sin^4 \theta \cos^2 \theta} \frac{\eta_{\bar{x}} \eta_{\bar{y}}}{\cos \theta} \\
 & + \epsilon_{\beta}^2 G \cdot \frac{1}{\sin \theta} \eta_{\bar{y}} \\
 & - \epsilon_H \epsilon_T \cdot \sin \theta \eta_{\bar{t}} (1 + \epsilon_H \eta)^{-1} \\
 & + \epsilon_H \epsilon_T \epsilon_R G \cdot \frac{1}{\sin \theta} \left[\frac{1}{2} \frac{\partial}{\partial \bar{t}} \left(\frac{\eta_{\bar{x}}^2}{\cos^2 \theta} + \eta_{\bar{y}}^2 \right) - \eta_{\bar{t}} \cdot \nabla^2 \eta \right] (1 + \epsilon_H \eta)^{-1} \\
 & + \epsilon_H \epsilon_T \epsilon_{\beta} G \cdot \frac{\eta_{\bar{y}} \eta_{\bar{t}}}{\sin^2 \theta \cos \theta} (1 + \epsilon_H \eta)^{-1} \\
 & - \epsilon_H \epsilon_R^2 G^2 \cdot \frac{1}{\sin^2 \theta} \frac{1}{\cos \theta} \left[\eta_{\bar{x}} \eta_{\bar{y}} \left(\frac{\eta_{\bar{x}\bar{x}}}{\cos^2 \theta} - \eta_{\bar{y}\bar{y}} \right) + \eta_{\bar{x}\bar{y}} \left(\eta_{\bar{y}}^2 - \frac{\eta_{\bar{x}}^2}{\cos^2 \theta} \right) \right] (1 + \epsilon_H \eta)^{-1} \\
 & - \epsilon_H \epsilon_R \epsilon_{\beta} G^2 \cdot \frac{\cos \theta}{\sin^3 \theta} \frac{\eta_{\bar{x}}}{\cos \theta} \left[\frac{\cos 2\theta}{\cos^2 \theta} \frac{\eta_{\bar{x}}^2}{\cos^2 \theta} + \eta_{\bar{y}}^2 \right] (1 + \epsilon_H \eta)^{-1} + O(\epsilon^3)
 \end{aligned}$$

**Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies**

Observations

Wave Interpretation
Eddy Interpretation
Attributes

**Linear vs
Nonlinear**

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Approximating the Complete Equation

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

To estimate the magnitude of all the terms, we need to

1. determine the magnitude of the $O(\epsilon^3)$ terms (there are no $O(\epsilon^4)$ terms)
 - ▶ These are finite and already computed.
2. determine the error in making a cartesian approximation.
 - ▶ Take a Taylor expansion of each term.

Example Values

Take an eddy,

- ▶ at latitude 45
- ▶ with a height of 10 cm
- ▶ length scale of 100 km
- ▶ time scale of 100 days ($L/T \sim$ non-dispersive speed)
- ▶ a geostrophic determined fluid speed of 6.7 cm/s

Then the coefficients are,

- ▶ $\epsilon_{\beta} G \cot \theta_0 \approx 7.2 \cdot 10^{-5}$
- ▶ $\epsilon_{T \in R} G \csc \theta_0 \approx 5.6 \cdot 10^{-6}$
- ▶ $O(\epsilon^3) \approx 3.9 \cdot 10^{-7}$
- ▶ $O(\text{cartesian}) \approx 1.1 \cdot 10^{-6}$
- ▶ etc.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Example Values

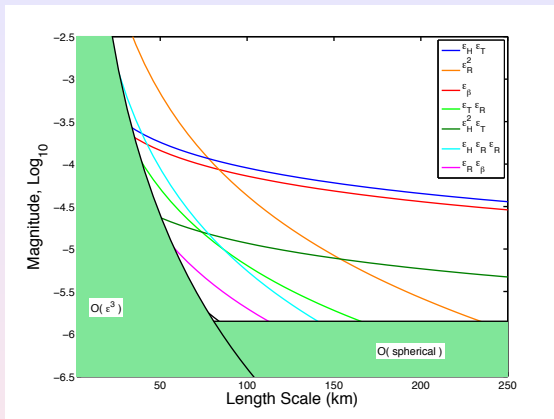


Figure: The magnitude of the terms in the complete equation are shown versus the length scale. The surface height is fixed at 10 cm, the latitude at 45 and the fluid velocity varies assuming $G = 1$. The ratio of the length scale to the time scale is fixed at 1.46 cm/s, slightly faster than the long wave Rossby phase speed at that latitude.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation

Eddy Interpretation

Attributes

Linear vs
Nonlinear

Numerical Setup

Monopole

Eddy Seeding

QG Monopoles

Shape

Trapped Fluid

Zonal & Meridional

Speed

Mini Conclusions

FP Equation

The FP Monopole

Balance Equations

Scales

Extended Theories

Mini Conclusions

Conclusions

Back to the FP Monopole

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Guiding Assumption

For the FP monopole to be a valid solution all five terms in the FP equation must be of greater magnitude than all the neglected terms.

- ▶ Given a latitude and length scale, the FP monopole solution determines the other values.

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

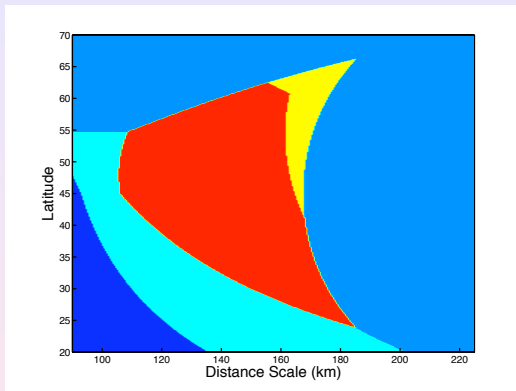
FP Equation

The FP Monopole
Balance Equations
Scales

Extended Theories
Mini Conclusions

Conclusions

Where the FP monopole is a valid solution



- ▶ FP equation terms dominate
- ▶ Extended FP theory
- ▶ Cartesian, $O(\epsilon^3)$, other simpler theories

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

What about other solutions?

- ▶ FP monopole is only *one* solution to the FP equation.
- ▶ FP equation may be the right equation for other parameters.

Determining an appropriate theory

We know solutions that we *want* to find; we know the scales that these eddies operate at.

- ▶ What is the right equation for the observed scales?

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

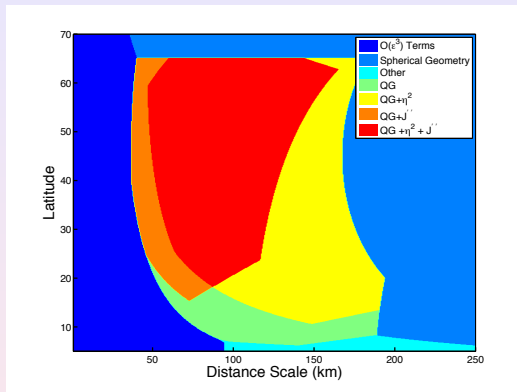
FP Equation

The FP Monopole
Balance Equations
Scales

Extended Theories
Mini Conclusions

Conclusions

An Appropriate Theory for the Observations



- ▶ 10 cm amplitude eddies (ϵ_H),
- ▶ in geostrophic balance,
- ▶ propagating at the non-dispersive phase speed.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales

Extended Theories

Mini Conclusions

Conclusions

FP + J'' theory

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

FP + J'' Equation

$$\frac{\partial}{\partial t} \left(\nabla^2 \eta - \left(\eta - \frac{1}{2} \eta^2 \right) \right) + \eta_x + \beta^{-1} J(\eta, \nabla^2 \eta) + J'' = 0$$

$$\text{where } J'' = \frac{1}{\sin^2 \theta} [\eta_x \eta_y (\eta_{xx} - \eta_{yy}) + \eta_{xy} (\eta_y^2 - \eta_x^2)].$$

- This new terms is an ageostrophic advection of height and stems from,

$$\epsilon_R \left(\frac{u}{\cos \theta} \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} \right)$$

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales

Extended Theories
Mini Conclusions

Conclusions

Mini Conclusions

- ▶ The FP monopole is unlikely to be a valid solution for our context.
- ▶ However, the FP equation and the extended FP equation *are* likely necessary corrections to QG.
- ▶ The biggest deficiency in QG stems from the small amplitude assumption.
- ▶ Technique can be applied to other scales!

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

FP Zonal Speed versus Amplitude

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

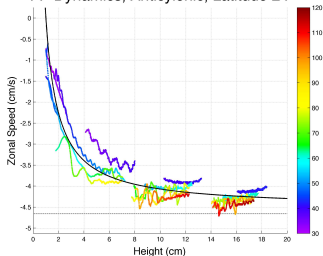
Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

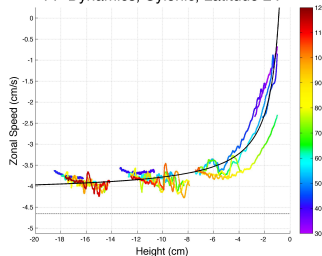
The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

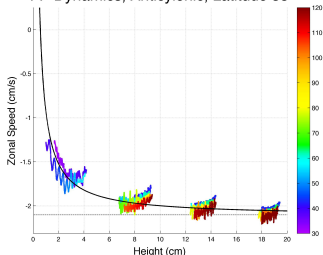
FP Dynamics, Anticyclonic, Latitude 24



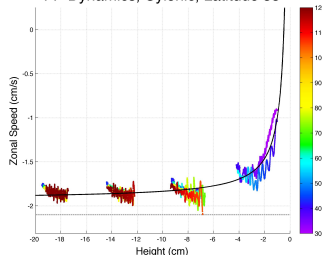
FP Dynamics, Cylonic, Latitude 24



FP Dynamics, Anticyclonic, Latitude 35



FP Dynamics, Cylonic, Latitude 35



Conclusions

- ▶ Linear Rossby wave dynamics fails to explain the satellite altimetry signal.
- ▶ Quasigeostrophy (QG) explains coherence, meridional deflection, spectra, propagation speed dependence, but fails the zonal propagation speeds.
- ▶ Extensions to QG such as the FP and extended FP theory may be required owing to the height nonlinearity.
- ▶ Group foliation can be used to find analytical solutions to nonlinear PDEs, such as the FP equation.

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Overview

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Primary Goal

Find new analytical solutions to the FP equation using group foliation.

Why?

- ▶ Nonlinear partial differential equations (PDEs) arise everywhere in physics.
- ▶ Numerical methods are increasingly used to solve such PDEs, but provide less confidence in understanding the physics of the solution.
- ▶ Analytical solutions to nonlinear PDEs very difficult, few generalized techniques.
- ▶ Group foliation is applicable to a wide range of PDEs.

Observations

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Eddy Interpretation
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Numerical Setup
Monopole
Eddy Seeding

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Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions

Example

Korteweg-de Vries

$$u_{xxx} - u_x + uu_x = 0$$

reduces to

$$aa_z = a - \frac{6}{25}z + \frac{2}{5}$$

2-D FP Equation in Polar Coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} - \psi + \frac{1}{2} \psi^2$$

reduces to

$$H_q = A_s H - A H_s,$$

$$H_s = -q^2 \left(A_q + A A_s + \frac{1}{q} A - s + \frac{1}{2} s^2 \right).$$

Theoretical
Approaches to
Understanding
Satellite Altimetry
Tracked Eddies

Observations

Wave Interpretation
Eddy Interpretation
Attributes

Linear vs
Nonlinear

Numerical Setup
Monopole
Eddy Seeding

QG Monopoles

Shape
Trapped Fluid
Zonal & Meridional
Speed
Mini Conclusions

FP Equation

The FP Monopole
Balance Equations
Scales
Extended Theories
Mini Conclusions

Conclusions