

# Numerical Experiments with the FP Equation

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Rossby Waves

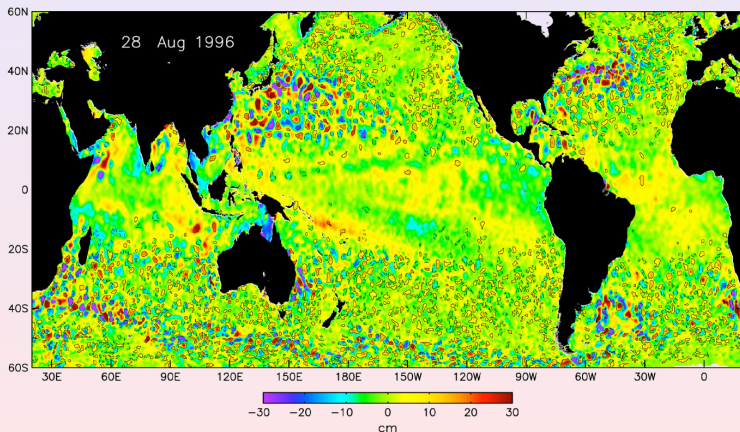
Theory &  
Observations  
FP Equation

Experiments

Linear  
Tan & Boyd Regime  
Ocean Regime  
Primitive Equation  
Model

# Observed Sea Surface Height Anomaly ( $\sim \psi$ )

- ▶ 18+ week coherent structures are contoured.
- ▶ 50,000 such features in 14 years of data.



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# Rossby Wave Theory

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- ▶  $\psi$  proportional to sea surface height.

## Primary Long Wave Balance

$$\psi_t - \psi_x = 0$$

Solved by any function  $\psi(x, t) = f(x + t)$ .

- ▶ Predicted propagation speed is too slow.
- ▶ No predicted structure.
- ▶ Linear theory also predicts some dispersion; more than actually observed.

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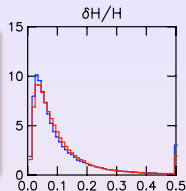
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## One Source of Nonlinearity

Distribution of observed surface displacement to equivalent depth suggests next to leading order terms matter.



Small surface displacement



Warm Water



Large thermocline displacement

Cold Water



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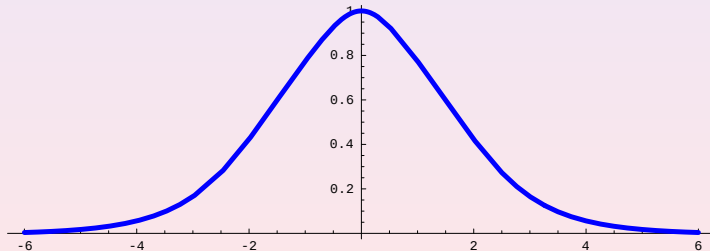
## Flierl-Petviashvili Equation

$$\frac{\partial}{\partial t} \left( \nabla^2 \psi - \left( \psi - \frac{1}{2} \psi^2 \right) \right) + \psi_x + J(\psi, \nabla^2 \psi) = 0$$

- ▶ Primary addition is the  $\psi^2$  term.
- ▶ Flierl also looked for eddy like structures, but a shear flow was his primary source of nonlinearity.
- ▶ Petviashvili considered the Great Red Spot on Jupiter.

# Hints of Interesting Solutions

- ▶ One known radial solution, found numerically.
- ▶ Does provide a speedup from linear theory.
- ▶ Positive sea surface height anomaly only (no waves of depression).



# How to proceed?

## Many Questions

How does this solution behave? Is it stable? Can a wave of depression evolve? Are there other solutions? How do the monopoles interact?

- ▶ Look for other analytical solutions.
- ▶ Solve FP equation numerically.
- ▶ Try solution in the more general primitive equations.

## Setup

Spectral differentiation with fourth order Runge-Kutta method for time stepping.

- ▶ Linear
  - 1. Monopole in linearized equation.
- ▶ Tan and Boyd Parameter Regime
  - 1. Isolated monopole.
  - 2. Inverted monopole.
  - 3. Interacting monopoles.
- ▶ Oceanic Parameter Regime (repeat above)
- ▶ Isolated Monopole in Primitive Equations

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## Linear Equation

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \psi = 0$$

(Linear.mov)

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## Nonlinear Equation, Isolated Monopole

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left( \psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

(IsolatedUnityJacobian.mov)

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## Nonlinear Equation, Inverted Monopole

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left( \psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

(InvertedUnityJacobian.mov)

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# Nonlinear Equation, Interacting Monopoles

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$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left( \psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

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(InteractionUnityJacobian.mov)

# Why their regime doesn't apply...

## FP Equation with Nondimensional Parameters

$$\nabla^2 \psi_t - a \cdot \psi_t + b \cdot \psi_x + c \cdot \psi \psi_x + k \cdot J(\psi, \nabla^2 \psi) = 0$$

- ▶ Four dimensionless parameters:  $a$ ,  $b$ ,  $c$  and  $k$ .
- ▶ Three free scales:  $\Psi$ ,  $T$ , and  $L$ .
- ▶ 4-3 = we're stuck with one scale dependent parameter.
- ▶ Tan & Boyd set the parameter to 1, equivalent to a baroclinic speed of 1400 MPH in our regime.

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## Nonlinear Equation, Isolated Monopole

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left( \psi + \frac{1}{2} \psi^2 \right) + 263 \cdot J(\psi, \nabla^2 \psi) = 0$$

(IsolatedOceanJacobian.mov)

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(InvertedOceanJacobian.mov)

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# Nonlinear Equation, Interacting Monopoles

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left( \psi + \frac{1}{2} \psi^2 \right) + 263 \cdot J(\psi, \nabla^2 \psi) = 0$$

(InteractionOceanJacobian.mov)

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# Primitive Equation Model

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## ROMS

Regional Ocean Modeling System. Solves three-dimensional equations, with far fewer assumptions than the FP Equation.

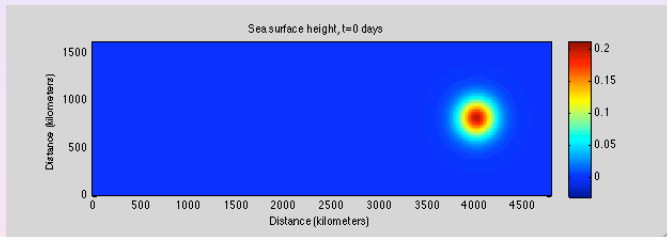
- ▶ Initialize model with density, velocity and free surface.
- ▶ FP monopole length scale, uniquely determine sea surface height and velocity field.
- ▶ Valid e-folding scale between 100 km and 160 km.

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# Conclusions

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- ▶ Be careful with your constants.
- ▶ FP Equation Model
  - ▶ Both FP parameter regimes have stable positive monopoles, but neither matches expected shape.
  - ▶ Boyd & Tan negative monopoles not stable, but ocean regime negative monopoles may be stable!
- ▶ Primitive Equation Model
  - ▶ FP monopole not stable.
  - ▶ Weaker amplitude monopole lasts for up to a year.