

Modeling the Flierl-Petviashvili Equation

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Introduction

Satellite observations, Rossby waves and the FP equation

- ▶ Satellite observations of mesoscale variability show many eddies, not just Rossby waves.
- ▶ The Flierl-Petviashvili (FP) equation may include the appropriate physics to explain the eddies.
- ▶ Analytical solutions are difficult to come by, so look to a numerical model for guidance.

Introduction

Rossby Waves

Theory &
Observations
FP Equation

Experiments

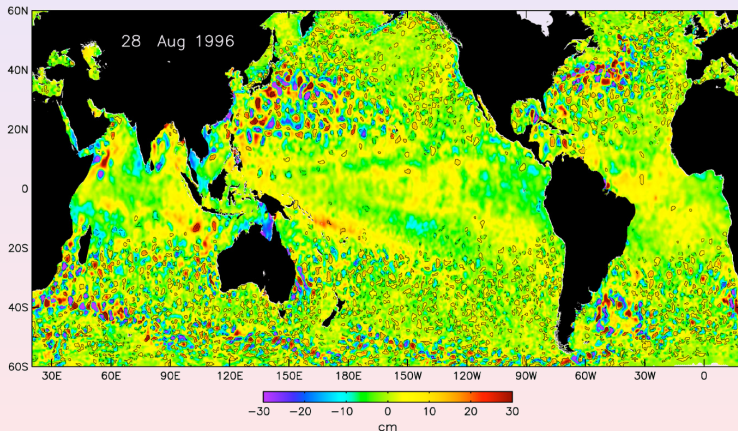
Tan & Boyd Regime
Ocean Regime

Observed Sea Surface Height Anomaly ($\sim \psi$)

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- ▶ 18+ week coherent structures are contoured.
- ▶ 50,000 such features in 14 years of data.



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Rossby Wave Theory

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- ▶ ψ proportional to sea surface height.

Primary Long Wave Balance

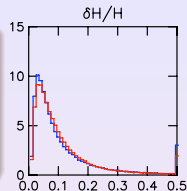
$$\psi_t - \psi_x = 0$$

Solved by any function $\psi(x, t) = f(x + t)$.

- ▶ Predicted propagation speed is too slow – or is it?
- ▶ Does not predict eddy structure.
- ▶ Linear theory predicts some dispersion; more than actually observed.

One Source of Nonlinearity

Distribution of observed surface displacement to equivalent depth suggests next to leading order terms matter.



Small surface displacement



Warm Water



Large thermocline displacement

Cold Water



FP Equation

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left(\psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

- ▶ Primary addition is the ψ^2 term.
- ▶ Flierl also looked for eddy like structures, but a shear flow was his primary source of nonlinearity.
- ▶ Petviashvili considered the Great Red Spot on Jupiter.
- ▶ Quick and dirty derivation....

FP Equation, Traveling Wave

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- ▶ Let $\xi = x - ct$ and rescale.

Traveling Wave reduced FP Equation

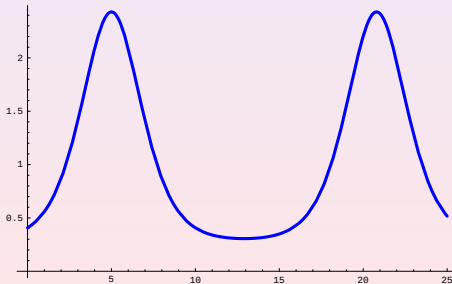
$$\frac{\partial}{\partial \xi} \left(\nabla^2 \psi - \psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

- ▶ **Special** solution exists. If you can solve $\frac{\partial}{\partial \xi} \left(\nabla^2 \psi - \psi + \frac{1}{2} \psi^2 \right) = 0$, then it follows that $J(\psi, \nabla^2 \psi) = 0$ and therefore you solve the equation.

- ▶ Make the equation one-dimensional by letting $\frac{d}{dy} = 0$, you get the KdV equation.

Korteweg-de Vries Equation

$$\psi_{\xi\xi\xi} - \psi_{\xi} + \psi\psi_{\xi} = 0$$



Radial FP Equation, Monopole Solution

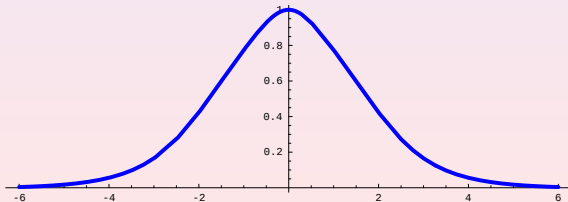
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Radial FP Equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) - \psi + \frac{1}{2} \psi^2 = 0.$$

- ▶ One known radial solution, found numerically.
- ▶ Provides a speedup from linear theory.
- ▶ Positive sea surface height anomaly only (no waves of depression).



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How to proceed?

Starting Observation

KdV solution and FP monopole suggest FP equation may have the correct physics for the observations.

- ▶ Find analytical solutions.
- ▶ Solve FP equation numerically.
- ▶ Try solution in the more general primitive equations.

Numerical Experiments

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Setup

Spectral differentiation with fourth order Runge-Kutta method for time stepping.

Primary Question

Does the FP equation produce eddy-like structures and under what conditions?

- ▶ Tan and Boyd Parameter Regime
 1. Isolated monopole.
 2. Inverted monopole.
 3. Interacting monopoles.
- ▶ Oceanic Parameter Regime (repeat above)

Nonlinear Equation, Isolated Monopole

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$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left(\psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

(IsolatedUnityJacobian.mov)

Nonlinear Equation, Inverted Monopole

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$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left(\psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

(InvertedUnityJacobian.mov)

Nonlinear Equation, Interacting Monopoles

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$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial \mathbf{x}} \left(\psi + \frac{1}{2} \psi^2 \right) + J(\psi, \nabla^2 \psi) = 0$$

(InteractionUnityJacobian.mov)

Why their regime doesn't apply...

FP Equation with Nondimensional Parameters

$$\nabla^2 \psi_t - a \cdot \psi_t + b \cdot \psi_x + c \cdot \psi \psi_x + k \cdot J(\psi, \nabla^2 \psi) = 0$$

- ▶ Four dimensionless parameters: a , b , c and k .
- ▶ Three free scales: Ψ , T , and L .
- ▶ 4-3 = we're stuck with one scale dependent parameter.
- ▶ Tan & Boyd set the parameter to 1, equivalent to a baroclinic speed of 1400 MPH in our regime.
- ▶ $k = \frac{f_0 R \tan \theta_0}{c}$.

Nonlinear Equation, Isolated Monopole

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$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left(\psi + \frac{1}{2} \psi^2 \right) + 263 \cdot J(\psi, \nabla^2 \psi) = 0$$

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(IsolatedOceanJacobian.mov)

Nonlinear Equation, Inverted Monopole

$$\frac{\partial}{\partial t} (\nabla^2 \psi - \psi) + \frac{\partial}{\partial x} \left(\psi + \frac{1}{2} \psi^2 \right) + 263 \cdot J(\psi, \nabla^2 \psi) = 0$$

(InvertedOceanJacobian.mov)

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Nonlinear Equation, Interacting Monopoles

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(InteractionOceanJacobian.mov)

Jacobian or ψ^2 ?

Do we really need the ψ^2 term? Or do we just need to get the scale on the Jacobian term correct?

A few quick tests with the trial and error method.

- ▶ FP monopole, ψ^2 , no Jacobian, $t = 0.05$.
- ▶ FP monopole, no ψ^2 , ocean Jacobian, $t = 0.05$.
- ▶ FP monopole, ψ^2 , ocean Jacobian, $t = 0.05$, $\epsilon = 0.01$.

Conclusion

The FP monopole needs the ψ^2 , but not the Jacobian.

Jacobian or ψ^2 ?

Do we really need the ψ^2 term? Or do we just need to get the scale on the Jacobian term correct?

- ▶ Gaussian monopole, ψ^2 , no Jacobian, $t = 0.05$, $\epsilon = 0.01$, $L = 4.0$, $h = 0.20$.
- ▶ Gaussian monopole, no ψ^2 , ocean Jacobian, $t = 0.05$, $\epsilon = 0.01$, $L = 4.0$, $h = 0.20$.
- ▶ Gaussian monopole, ψ^2 , ocean Jacobian, $t = 0.05$, $\epsilon = 0.01$, $L = 4.0$, $h = 0.20$.

Conclusion

The Gaussian monopole needs both the ψ^2 and not the Jacobian.

Dipole Interaction

How do two monopoles of opposite sign interact?

- ▶ Dipole, Strong Negative
- ▶ Dipole, Offset

Conclusion

Cyclones get sent poleward, monopoles remain intact.

White Noise

Initialized with low frequency white noise, evolved for one decade.

- ▶ Experiment 001

Conclusion

Both positive and negative monopoles evolve and persist over a range of scales.

Limitations

Over what scales is this numerical model valid? Preliminary...

- ▶ Assuming 100-150 km length scales, models breaks down at latitudes poleward of about 50 degrees due curvature and NNLO interface displacement.
- ▶ A latitudes equator-ward of 20 degrees NLO ageostrophic terms apply.

For this model,

- ▶ Speeds as a function of latitude, height.
- ▶ Shape of evolved monopoles.
- ▶ Fluid transport properties.
- ▶ Valid scales.

For ROMS,

- ▶ Evolution of 2D reduced gravity model.
- ▶ Stable monopoles? What scales? What shapes?